Tehnical and scientifical final report – General

We proposed (**Ob. 1.1**) finite elasto-plastic models, with a second order measure of deformations characterized by the set of plastic distortion and plastic connection. The models are developed for materials with inhomogeneities such as continuously distributed dislocations or locally damaged zones. The theory rises from the constitutive models with couple stresses and non-Riemannian connection and uses the differential geometry concepts like torsion and curvature of the connection and energetic arguments. The thermomechanic restrictions are provided based on the requirement to have the temporal variation of the free energy density less than or equal to the internal power (i.e. the imbalanced free energy condition), expended during the isothermal process. The physical forces, i.e the non-symmetric Cauchy stress tensor and stress momentum, are power conjugated with the velocity gradient and to its spatial gradient, respectively. The material forces acting on the inhomogeneities (say on dislocations and on microdefects) are defined by the microstress and by the microstress momentum, which are power conjugated to the rate of plastic distortion and to its gradient. Different constitutive representations can be adopted for the free energy density, having in mind the decomposition theorems applied to the plastic connection. The role of the plastic torsion and the non-local effects that can be predicted by the models are also discussed.

For **Ob. 1.2.** Shape memory alloys – elastic and thermoelastic non-monotone model versus rate-type Maxwellian model. Comparing exact elastic solutions with numerical solutions for the rate-type model in impact problems of flexible extensible strings (isothermal case) and of two thin bars (thermo-mechanical case). Nucleation, propagation of phase boundaries, interactions of shock waves, hysteresis and damping effects are well described by the rate-type model, when compared to laboratory experiments.

Initiation of a rigorous mathematical theory for bipotentials (used in connection to constitutive laws). A necessary and sufficient criterion for a constitutive law to be expressed by a bipotential and a way to construct that bipotential. Extention of these results to a convex hull of bipotentials with application to the bipotential corresponding to the Coulomb friction law.

The functionally graded materials (**Ob. 1.3.**) have been studied intensively in the last years (see, for example, Flavin (1995); Pindera et al. (1997)). As an example of such materials we can give the class of elastic materials with continuous inhomogeneity. The study of the inhomogeneity of the material upon the decay rate within the context of Saint-Venant principle represents a direction of research of great importance from the technological and practical point of view. We have to mention that, for a homogeneous material occupying an arch-like region, a Saint-Venant principle with respect to the growth of the polar angle has been established by Flavin (1992), and Flavin and Gleeson (2003), under a very strong restriction upon the dimensions of the region. The study of the spatial behavior in elastic cylinders is often based on energetic measures or cross-sectional measures associated with the solution. Only the paper by Flavin, Knops and Payne (1989) uses a measure in terms of the displacement in the case of isotropic and homogeneous elastic materials. In this project we developed this idea for the whole class of general anisotropic elastic.

In the international studies on the porous solids (**Ob. 1.4.**) an important direction of research is that of study of the mechanical behaviour of the thin bodies (like plates and shells) made of elastic materials with voids. Using the Nunziato-Cowin theory for the study of porous elastic plates, a monograph, which includes the thermomechanics behaviour of these, has been written in this project. On the otherhand, we consider the Saint-Venant's problem for right cylinders made of an inhomogeneous anisotropic elastic material with voids. For the treatment of this problem, we resort to the method used by Chirita (1992) for the study of Saint-Venant's problem in linear viscoelasticity. We study the possibility to reduce the problem to some generalized plane strain problems, which are more tractable. For this, we point out two classes of semi-inverse solutions in the set of solutions of Saint-Venant's problem that may be expressed in terms of a plane displacement. These classes are relevant in obtaining a semi-inverse solution to the relaxed Saint-Venant's problem. We use the results obtained in the anisotropic case to solve the extension, bending, torsion and flexure problem of isotropic porous elastic circular cylinders.

In the last decades, a great effort was made in order to develop theories of mixture (**Ob. 1.5.**) of various types of materials. The study of such mixtures is very important from a technological point of view. Using the theory developed by Iesan (2004), we established an existence result of solution and we study the asymptotic echipartition of energy. From the method used by Horgan (1984) for parabolic equations and from the generalized method of Quintanilla (2001) for mixture between parabolic and hyperbolic equations, we extend the result of Pompei si Scalia (2002) for spatial behaviour of thermoelastic mixtures of two elastic solids. Moreover, we propose a new theory to describe the behaviour of the mixtures of two composed thermoelastic materials with internal structure.

Concerning the **Objective 2.1**. the main results which were obtained are the following: 1) Existence of periodic solutions for non-autonomous second order systems and for evolution complementarity systems (by using a method of guiding functions). 2) Existence and multiplicity results for nonlinear equations involving duality mappings. 3) The structure of the solution set for a class of nonlinear equations involving a duality mapping. 4) Non-existence results of weak solutions for p-Laplacian (Dirichlet problem) via a generalized Pohosžaev identity. 5) Brouwer degree and applications (monograph accomplished).

Convergence of some multiplicative and additive Schwarz (**Ob. 2.2.**) methods for inequalities which contain contraction operators. The problem, stated in a reflexive Banach space, generalizes the well-known fixed-point problem in the Hilbert spaces. Error estimation theorems are given for three multiplicative algorithms and two additive algorithms. These algorithms are in fact Schwarz methods if the subspaces are associated with a decomposition of the domain. For the one- and two-level methods in the finite element spaces, convergence rates may be written as functions of the overlapping and mesh parameters. Above results are used to obtain the convergence rate of the Schwarz method for other types of inequalities or nonlinear equations: convergence and estimate of the error of one- and two-level Schwarz methods for some inequalities in Hilbert spaces which are not of the variational type, for the Navier–Stokes problem. Conditions of existence and uniqueness of the solution for all problems considered.

In the framework of **Objective 2.3**. it was continued the study of operatorial equations which can be solved using variational methods. Using the concept of τ -uniform convexity it was considered a unitary frame for a large series of variational methods which can be used in the study of some special classes of operatorial equations. In the development of least-squares method, establishing the coercivity of the corresponding bilinear form is a crucial step and there has been big improvement in the techniques to prove coercivity. It was performed a deep study of the coercivity.

In this project we concentrated (**Ob. 2.4.**) on the numerical approximation of some evolution operators used in physics and mechanics of continua. The numerical methods refer to Boltzmann equations and generalized Navier Stokes equations. To approximate the Boltzmann equation in the entire space a suitable discretization scheme was elaborated. This scheme allows to obtain uniform error estimations with respect to the time and space variables. The problem of developing rigorous numerical algorithm for the kinetics of the reacting gas has been also considered. We have provided a rigorous scheme, convergent (in measure) in some probabilistic sense, which solves numerically a weak form of generalized Boltzmann model with chemical reactions. Concerning the generalized Navier Stokes equations we have introduced a new numerical algorithm for computing the flow of a class of pseudo-plastic fluids. Such a model, with the viscosity depending on the strain rate, is frequently used as a mechanical model of the blood flow. The method uses the finite volume technique for space discretization and a semi-implicit two steps backward differentiation formulae for time integration. As primitive variables the algorithm uses the velocity field and pressure field. In this scheme one uses quadrilateral structured primal-dual meshes. The velocity and the pressure fields are discretized on the primal mesh and the dual mesh respectively. By a proper definition of the discrete derivative operators we have been able to prove a Hodge decomposition formula. Based on it we can calculate independently the velocity and pressure. A certain advantage of the method is that the velocity and pressure can be computed without any artificial boundary conditions and initial data for the pressure. Based on the numerical algorithm we have written a numerical code. We have also performed a series of numerical simulations.

Some results have been obtained concerning the free boundary flow (**Ob. 2.5**). It was considered Helmholtz's model for potential plane flow of a fluid past an obstacle and we reduced the free boundary value problem to the study of the fixed points of a nonlinear operator. In the case of the circular symmetric obstacle, one obtained a nonlinear integral equation which was solved numerically using the successive approximations method. It was also investigated the study of the cracks in pre-stressed elastic composite materials. We had in view the anti-plane problem. One utilized the boundary conditions imposed on the crack in order to reduce the problem to a Hilbert-Riemann problem which leads in the end to a linear complex non-homogeneous differential equation. The differential equation was solved numerically.

For the **objective 2.6. a**n alternative approach to the study of dynamical systems is using the Birkhoffian formalism (a larger view than Lagrangian or Hamiltonian formalism). The concepts and the direct theorems of stability in the sense of Liapunov, within this framework, are considered. The Liapunov-type functions are constructed for linear and non-linear LC and RLC electrical networks, to prove stability under certain conditions.

In the framework of objectives (**3.1 and 3.2.)** we employed the models of second grade, third grade, Bingham, Maxwell and Oldroyd-B fluids. We have investigated various problems and we have proposed a friction law at solid boundaries in the hope to explain some defects appearing in particular flows in real geometries. These fluid models have been employed to the modeling of real phenomena like: Stokes' flows, flows in orthogonal rheometers, shock expansion of a spherical cavity in a medium which behaves like a viscoplastic material with locked hydrostat. It has been potted into evidence "strange" behaviors of some of these fluids (third grade) like: secondary motions in elliptic pipes and the propagation of acceleration waves. The dissipation, the power due to the shear stress at the wall, the boundary layer thickness and changing of the kinetic energy with time, corresponding to the Rayleigh-Stokes problem for a second grade fluid and Maxwell's fluid have been calculated for different motions. Also, the energetic balance in the Rayleigh-Stokes problem for a Maxwell fluid was studied for several initial and/or boundary conditions. Exact and approximate expressions for the power due to the shear stress at the wall, the dissipation and the boundary layer thickness are established for the unsteady flow of an Oldroyd-B fluid due to an infinite plate subject to a constant shear stress. In order to solve these problems various mathematical techniques have been employed: asymptotic expansions, exact solutions, integral transform techniques etc.

Secondary oil recovery using a polymer with surfactant properties as a second immiscible fluid, was the pourpose of **objective 3.3.** For the multi-layered Hele-Shaw and darcy models a “sharp interface” exists between the fluids; one gives some results concerning the linear stability of this interface: a new formula of the growth constant of perturbation, a stability criterion in terms of the viscosity ratio of the considered fluids, a strategy for minimizing the instability. For the Buckley-Leverett model: a stability analysis related with a “constitutive” function has been done.

Study of a fluid-structure interaction problem, the non periodic case. Non stationary flow of a viscous fluid in a thin rectangle with an elastic membrane as the upper part of the boundary. An asymptotic solution is proposed by using a boundary layer method. An error estimate is obtained.

Degenerate parabolic equations in nondivergence form (used in compressible fluid flow through porous media – **Ob. 3.4.**) of the type $\partial t\ u = a(\delta(x))u^n\ (\Delta u + \lambda g(u))$ in $\Omega\times (0,1)$, where Ω in R(N), ($N \geq 1$) is a smooth bounded domain, $\delta(x) = \mathrm{dist}(x, \partial\Omega)$, $\lambda > 0$, $p \geq 1$, and g is either a nondecreasing function having a sublinear growth or $g(u) = u$. Under some suitable assumptions on g, a, and λ, one establishes the existence and uniqueness of a classical solution and one determines its asymptotic profile as $t \to \infty$. If $g(u) = u$ a blow-up result as λ approaches the first eigenvalue $\lambda 1$ of the Laplace operator $-\Delta$ is found.

Recently, Eringen (2003) proposed a continuum theory for the mixture of a micropolar elastic solid and a viscous micropolar fluid. A large class of engineering materials like soils with grains, rocks, granular materials, sand and dirty fluids may be modeled, more realistic, with this theory. For the initial boundary value problem from this theory we obtained the results regarding the uniqueness, existence and continuum dependence of solution with respect to data. We also studied the asymptotic echipartition of energy. This results show that the model is well posed from mathematical point of view (**Ob. 3.5.**).

Director
Prof. dr. Sanda Tigoiu

Published and accepted papers

Papers

1) S. Cleja-Tigoiu, O.Cazacu, V.Tigoiu, ”Dynamic expansion of a spherical cavity within a rate-dependent compressible porous material”, International Journal of Plasticity, 24, 5(2008), 775-804;
2) S. Cleja-Tigoiu, ”Material forces in finite elasto-plasticity with continuously distributed dislocations”, International Journal of Fracture, 147, 1-4(2007), 67-81;
3) A. Carabineanu, ”Self-propulsion of oscillating wings in incompressible flow”, Int.J.Numer. Meth. Fluids (acceptata la 21 Martie 2007);
4) A. Carabineanu, ”The free-boundary flow past an obstacle. Qualitative and numerical results”, Carpathian J. Math. 23(2007), No.1-2, 53-62;
5) G. Dinca, P. Matei, ”Variational and topological methods for operator equations involving duality mappings on Orlicz-Sobolev spaces”, Electronic Journal of Differential Equations, Vol2007(2007), No.93, pp.1-47(http://ejde.math.txstate.edu)
6) J. Zierep, C. Fetecau, ”Energetic balance for the Rayleigh-Stokes problem of a second grade fluid”, Int. J. Eng. Sci. 45 (1), 155-162, 2007;
7) J. Zierep, C. Fetecau, ”Energetic balance for the Rayleigh-Stokes problem of a Maxwell fluid”, Int. J. Eng. Sci. Sci. 45 (2-8), 617-627, 2007;
8) S. Cleja-Tigoiu, Thermomechanics of materials with structural nonhomogeneities, Mechanics & Mechanisms of Finite Plastic Deformation, Eds. A. Khan, B. Farroch, ISBN: 0-9659463-8-X;
9) G. Dinca si P. Jebelean, Apriori estimates for the vector p-Laplacian with potential boundary conditions, Archiv der Mathematik 90 (2008), 60-69;
10) G. Dinca si M. Rochdi , On the structure of the solution set for a class of nonlinear equations involving a duality mapping, Topological Methods in Nonlinear Analysis, Vol. 31, No. 1 (2008), 29-49;
11) G. Dinca si J. Cringanu, Multiple solutions for a class of nonlinear equations involving a duality mapping, Diff. Int. Equations, Volume 21, Numbers 3-4, 2008, 265-284;
12) G. Dinca si P. Matei, Multiple solution for operator equations involving a duality mapping on Orlicz-Sobolev spaces, Diff. Int. Equations, Volume 21, Numbers 9-10, 2008, 891-916;
13) G. Dinca si P. Matei, Infinitely many solutions for operator equations involving duality mappings on Orlicz-Sobolev spaces, Topological Methods in Nonlinear Analysis (accepted);
14) G. Dinca si P. Matei, Multiple solution for operator equations involving duality mappings on Orlicz-Sobolev spaces via the Mountain Pass theorem, Rev.Roum. Math.Pures et Appl. (accepted);
15) G. Dinca, A remark on the existence of a triangle with prescribed angle bisector lengths, Bull. Belg. Math. Soc. Simon Stevin (accepted);
16) E.M. Craciun, A. Carabineanu, N. Peride, Fracture analysis of an oblique crack propagation in a pre-stressed glass-epoxy composite, KeyEngng. Materials, Vol. 385-387 (2008), pp. 733-736;
17) C. Pricina, E.C. Cipu si V. Tigoiu, On the flow of a third grade fluid in an orthogonal rheometer, Proc. Of the WASET, 30 (2008), pp. 226-230;
18) V. Tigoiu, Non-standard properties for a class of non-newtonian fluids, Proc.XXXI-th Conference on Solide Mechanics, Chisinau 27.09.-29.09.2007, pp. 358-362;
19) S. Cleja-Tigoiu, Incompatibility of plastic deformation, Proc. XXXI-th Conference on Solide Mechanics, Chisinau 27.09.-29.09.2007, pp. 324-328;

20) S. Cleja-Tigoiu, Thermomechanics of materials with structural nonhomogeneities, modeled by non-zero torsion or curvature., trimis spre publicare la International Journal of Plasticity;
21) S. Cleja-Tigoiu, D. Fortune and C. Vallee, Torsion Equation in Anisotropic Elasto-Plastic Materials with Continuously Distributed Dislocations, Math. and Mech. of Solids 2008, doi:10.1177/1081286507079157;
22) S. Cleja-Tigoiu, Model of elasto-plastic material with continuously distributed dislocation (Eulerian description), Analele Universitatii din Bucuresti, matematica, LVI(2007), 269-288;
23) W. Akhtar, C. Fetecau, V. Tigoiu si C. Fetecau, Flow of a Maxwell fluid between two side walls induced by a constantly accelerating plate, ZAMP, DOI 10.1007/00033-008-7129-8.
24) C. Faciu, A. Molinari, On impact induced propagating phase boundaries. Thermal effects, in:"Multiphase and multi-component materials under dynamic loading", W.K. Nowacki and Han Zhao eds., IPPT PAN and LMT Cachan, Proc. of EMMC 10, 2007, 27-40
25) C. Faciu, M.Mihailescu-Suliciu, Wave propagation and phase nucleation in phasetransforming strings, in : "Nonlinear Dynamics of Composite and Smart Structures", Proc. of Euromech Colloquium 498, J.Warminski, M.P.Cartmell and J.Latalski eds., Lublin 2008, 154-158
26) M. Buliga, G. de Saxce, C. Vallee, Existence and construction of bipotentials for graphs of multivalued laws, J. Convex Analysis, 15 (2008), 87-104
27) M. Buliga, G. De Saxce, C. Vallee, Non maximal cyclically monotone graphs and construction of a bipotential for the Coulomb's dry friction law, arXiv: 0802.1140v2 [math.FA], 14 Mar 2008)
28) L. Badea, Schwarz methods for inequalities with contraction operators, J. Comput. And Appl. Math., 215 (2008), 196-219)
29) D. Ionescu-Kruse, Liapunov's direct method for Birkhoffian systems: applications to electrical networks, J. of Geometry and Physics, 57, 2007, 2213-2228
30) G. Pasa, Stability results for several models of secondary oil recovery, Mathematical Reports, 10(60) 2/ 2008 , 169-183.
31) G. Panasenko, R. Stavre, Asymptotic analysis of a non-periodic flow in a thin channel with visco-elastic wall, Networks and Heterogeneous Media, 3, sept.2008, 1-20
32) M. Ghergu, On the global solutions to a class of strongly degenerate parabolic equations, Nonlinear Analysis, 2008 (in press)
33) S. Chirita si M. Ciarletta, Spatial estimates for the constrained anisotropic elastic cylinder, Journal of Elasticity, 85 (2006), pag. 189-213.
34) S. Chirita, Spatial behavior in the strongly elliptic anisotropic thermoelastic materials, Journal of Thermal Stresses, 30 (2007), pp. 1-15.
35) S. Chirita, On the spatial evolution in anisotropic thermoelasticity. The Seventh International Congress on Thermal Stresses THERMAL STRESSES 2007, June2007, Taipei, Taiwan, 97-100.
36) Ciro D'Apice si Stan Chirita, End effects for a generalized biharmonic equation with applications to functionally graded material, Journal of Mathematical Analysis and Applications, 342 (2008), 585-600.
37) M. Brsan, Inequalities of Korn's type and existence results in the theory of Cosserat elastic shells, Journal of Elasticity, 90 (2008), 227-239 .
38) C. Gales, On the asymptotic spatial behaviour in the theory of mixtures of thermoelastic solids, International Journal of Solids and Structures, 45 (2008), 2117-2127.
39) C. Gales, Some results in the dynamics of viscoelastic mixtures, Mathematics and Mechanics of Solids, 13, 2 (2008), 124-147 .
40) I. D. Ghiba, Some uniqueness and continuous dependence results in the micropolar mixture theory of porous media, International Journal of Engineering Science, 44 (2006), 1269-1279.

41) I.D. Ghiba, Existence and Uniqueness Results in the Micropolar Mixture Theory of Porous Media, in: Applied Analysis and Differential Equations, (editori O. Carja si I.Vrabie), World Scientific, New Jersey, 2007, 139-152.
42) I.D. Ghiba, Semi-inverse solution for Saint-Venant's problem in the theory of porous elastic materials, European Journal of Mechanics A/Solids, 27 (2008) 1060-1074.
43) I.D. Ghiba, Problema lui Saint-Venant pentru cilindri circulari in teoria materialelor elastice poroase izotrope, Lucrarile Editura Universitatii "Alexandru Ioan Cuza" , Iasi 2007, 99-111
44) I.D. Ghiba, Asymptotic partition of energy in micropolar mixture theory of porous media, Meccanica, doi 10.1007/s11012-008-9145-3, in curs de aparitie;
45) C.P. Grunfeld and D. Marinescu, On a convergent numerical method for nonlinear Boltzmann type models, Procceding of ICNPAA-2008, Cambridge Scientific Publishers, London (va apare).

Books

1) M. Birsan, Deformarea placilor elastice poroase: Un studiu matematic, Editura Matrix Rom, Bucuresti, 2007.
2) Ctin. Fetecau, C. Fetecau and V. Tigoiu, "Flows of non-Newtonian Fluids of Rate and Differential Type", 2009, Ed. Academiei Romane (va aparea).