1. PROPOSAL'S TITLE IN FULL

MATHEMATICAL MODELING IN CONTINUUM MECHANICS. APPLICATIONS IN MATERIAL SCIENCES. (M4_CONT)

1.1 AT: 11. Basic sciences: mathematics, physics, chemistry, biology.

1.2 Abstract

1.2.1 Motto. „A theory is a mathematical model for an aspect of nature. One good theory extracts and exaggerates some facets of the truth. ... A theory cannot duplicate nature, for if it did so in all respects, it would be isomorphic to nature itself and hence useless,..." – C.Truesdell, „Fundamental's of Maxwell's Kinetic Theory of a Simple Monatomic Gase" (cu R.G. Muncaster).

1.2.2 Motivation: The four partners in this project have scientific collaborations since long time ago. Thus: CO, P1 and P3 collaborate through: **a)** Co-organizers of the scientific seminary „Mechanics of Deformable Media" – more than 7 years; **b)** Co-organizers of several national and international conferences - The 5-th Int. Conf. „Geometry, Continua and Microstructures", Sinaia 2001; Int.Conf. „New Trends in Continuum Mechanics", Constanta 2003; „The National Conference – Caius Iacob – of Fluid Mechanics and its Technical Applications", Bucharest 2001, 2005; **c)** Common contributions to the series „Current Topics in Continuum Mechanics", Romanian Academy Publ., I – 2002, II – 2003, III – 2006; etc.Some P2 team members contributed to the above seminary with scientific communications or Ph.D. theses. **The present proposal** also aims to strengthen the scientific relations between teams that approach different aspects in Continuum Mechanics, starting from modeling and ending with numerical results (compared to experimental data), in order to prepare the future participation of the whole team to the FP7-programme (toghether with other European teams already scientifically connected with our teams).

1.2.3 Abstract

The Project is **structured on three main themes**, having several objectives each. The aim is: **obtaining new mathematical models for** different classes of **solids, fluids and mixtures; testing these models and constructing efficient mathematical and numerical methodes** to solve the appropriate problems; **obtaining efficient algorithms.** **Theme 1.** One will construct and study models for: a) anisotropic elasto-plastic materials and shape memory aloys; b) Visco-plastic materials with thermo-mechanic cupling and phase transformations, modeling smart materials, TRIP steels, etc.; c) elastic materials with continuous inhomogeneity („functionally graded"), porous materials and visco-elastic porous mixtures, taking into account micro-rotations, dilatations and contractions as well. **Theme 2.** a) One generalizes the duality application on Orlicz-Sobolev spaces with applications in non-linear elasticity (boundary value problems with operators that generalize the p-Laplacian). b) One will ellaborate different numerical methods: multigrid methods for non-variational inequalities (for nonlinear problems); convex and non-convex variational methods for nonlinear elasticity, visco-plasticity (geomaterials flow); approximations schemes for evolution operators (effectiv algorithms for diffusion type operators); numerical solutions for hypersingular integral equations in aerodynamics (a helix foil moving on a helical surface). This theme also contains the geometric study of the birkhoffian formalism with applications in the dynamics of mechanical systems. **Theme 3.** One will sudy properties of some classes of newtonian and non-newtonian fluids with applications in industrial processing and biomechanics. Thus: a) one studies nonlinear differential and integral models by means of two types of admissible cinematical fields with applications to polymer extrusion; b) one studies the energetical equilibrium for second grade, Maxwell and Oldroyd-B fluids in the Rayleigh-Stokes problem; c) one compares stability in different models (Hele-Shaw, Buckley-Leverett and saturation model) in the secondary oil recovery problem; d) one will use asymptotical methods for the non-periodic motion of a viscous fluid in a thin pipe with a visco-elastic boundary – a model for the blood motion through an artery; e) a qualitative study of turbulence models for a solid-fluid mixture.

2. PRESENTATION OF NATIONAL AND INTERNATIONAL CONTEXT WITHIN THE MENTIONED THEMATIC FIELDS:

Continuum Mechanics has proved an essential and powerful instrument of phenomenological research with wide applicability in technological research and production, from the spatial and car industries to metallurgy and medicine. The teams in this Project have, separately or in collaboration, accumulated a wide experience in the domain and joining these experiences and work methods under a common project, will allow the approach of new problems with significant theoretical results, also having a large applicability. The proposed themes contain **a series of theoretical approaches** in **modelling** various classes of materials (solids, fluids, mixtures) and modern **mathematical and numerical techniques** in order to solve the appropriate problems, on one hand. On the other hand, they also contain **effective methods to solve the problems** (up to numerical results and finally, the comparison of the model prediction to experimental data). **The proposed themes are: 1) New characterisations of thermo-elasto-viscoplastic materials with structural in-homogeneities. 2) Functional, geometric and numerical models in continuum mechanics. 3) New models and approaches for motions of some classes of newtonian and non-newtonian fluids.** The subjects considered in these project themes are of a real interest in the national and international research work and rely on the rich national and international cooperation experience of our team members. Young researchers, Ph.D. and master students will also be involved in the research of these themes. We shortly present in the following, the present situation in the considered domains. **Theme 1.** ■ A macroscopic modelling of materials with inelastic (irreversible) properties has top take into account the specific microstructure, essentially influencing their behaviour. There are two directions mainly developed worldwide, elasto-plastic constitutive theories with finite deformations, of first order and of second order. Significant results have been obtained in finite elasto-plasticity with only first order effects, for materials with crystalline structure, by consistent theories: Mandel (1971), Teodosiu (1970), Rice (1971), Kroner (1992), Rajagopal and Srinivasa (1995). The internal state mechanism and variables are defined by evolution equations. The different stress measures included in "driving forces" models, are used to describe at macroscopic level, that the plastic (irreversible) deformation starts only when certain critical values are reached. Here are to be mentioned the works of: Kratochvil (1971), Mandel (1972), Loret (1983), Maugin (1994), Halphen and Nguyen (1975), Van der Giessen (1989), Dafalias (1985), Simo (1998), Miehe (1997), etc. In our country a possible axiomatic reconstruction of a model for elasto-plastic materials with crystalline structure has been developed by Cleja-Tigoiu (1990-I,II), Cleja-Tigoiu and Soos (1990). They generalized some results in the constitutive frame of anisotropic materials and determined rate type constitutive equations in anisotropic finite elasto-plasticity. They also discussed different plastic models with continuous dislocations and introduced a new concept of material symmetry, distinct of Noll's one (1967). (see S. Tigoiu CV). ■ In the last decades new materials with special thermo-mechanical properties have been produced and used due to their important technological applications. Among them we find shape memory alloys and some classes of steels exhibiting the TRIP effect – manifested by a significantly increased plasticity due to a phase change. A deformation of these materials under different thermo-mechanical conditions implies the coupling of several mechanisms and phenomena: mechanical and thermal phenomena, phase transformation and rate type mechanical aspects. Mathematicians are more and more involved in modelling and investigating these properties in different research centres worldwide. The Romanian school of mechanics has, starting with the papers of I.Suliciu and Soos of the nineties, a strong tradition in modelling phase transformation phenomena. On the other hand, the research group from IMAR has since long ago good contacts and collaborations on this topics with the Universite de Metz, Laboratoire de Physique et Mecanique des Materiaux (Prof. A. Molinari) and the Institute of Fundamental Technological Research of Warsaw (Prof. W. Nowacky); their results in modeling phase transformation for shape memory alloys are internationally appreciated (see Faciu's and M.Suliciu's CV). We also mention the collaborations in the frame of the programs EURROMMAT (IMAR-UE centre of excellence) and EGIDE ECO-NET (France-Poland-Romania collaboration program financed by France). Thus, the combined work of mathematicians, engineers and experimental teams leads to an interdisciplinary character of the research. ■ The functionally graded materials have been studied intensively in the last years (see, for example, Flavin (1995); Pindera et al. (1997)). As an example of such materials we can give the class of elastic materials with continuous inhomogeneity. The study of the inhomogeneity of the material upon the decay rate within the context of Saint Venant principle represents a direction of research of great importance from the technological and practical point of view. Some spatial decay estimates have been established by Flavin (2003). We have to mention that, for a homogeneous material occupying an arch-like region, a Saint Venant principle with respect to the growth of the polar angle has been established by Flavin (1992), and Flavin and Gleeson (2003), under a very strong restriction upon the dimensions of the region. Such a restriction was completely avoided by Chirita (2005). A principle of Saint Venant type with respect to the growth of the polar distance was obtained by Chirita and D'Apice (2005). On the other hand, the study of the spatial behavior in elastic cylinders is often based on energetic measures or cross-sectional measures associated with the solution. Only the paper by Flavin, Knops and Payne (1989) uses a measure in terms of the displacement in the case of isotropic and homogeneous elastic materials. It should be interesting to develop this idea for the whole class of general anisotropic elastic materials (see, for example, the list of publications by Chirita). ■ A description of the mathematical model of porous media (with voids), with a complete bibliography) is furnished by the book written by Iesan (2004). In the international studies on the porous solids an important direction of research is that of study of the mechanical behaviour of the thin bodies (like plates and shells) made of elastic materials with voids. Thus, the approach for porous plates of Mindlin type was introduced and investigated by E. Scarpetta (2002) and M. Birsan (2003). In this connexion one requires the study of the corresponding initial boundary value problems and establishing the basic properties of the solutions. The deformation of a prismatic cylinder made of a classical elastic material was recently considered by V. V. Meleshko (2003), while the deformation of the porous cylinder was studied by F. Dell'Isola and R. C. Batra (1997), where the flexure problem is treated in an erroneous manner. ■ In the last decades, a great effort was made in order to develop theories of mixture of various types of materials. The study of such mixtures is very important from a technological point of view. Recently, Iesan (2004) has developed a theory of viscoelastic mixture by taking into account the effect of porosity. Moreover, Iesan (2004) has established some results concerning the uniqueness and continuous dependence of solutions with respect to the external data. It should be continued the study of the qualitative properties of this important model. In 1990 Eringen proposed a theory for describing the deformation of elastic solids in which each point can obey to dilatations or contractions. More recently, Eringen (2003) developed a theory for describing a mixture of a micropolar solid and a micropolar viscous fluid. A more complete theory of porous media should involve the microrotations, as well as the possibility that each point to have proper dilatations and contractions. **Theme 2.** ■ *A)* The duality mapping is one of the most remarkable examples of monotone surjective operators (Minty, Browder theory), on reflexive Banach spaces. *B)* Many remarkable differential operators are duality mappings. In this way, the theoretical surjectivity results obtained for duality mappings can be translated in the existence results for some boundary problems attached to those differential operators, for example the p-Laplacean. *C)* The p-Laplacean operator arises in mechanical models: non-Newtonian fluids, the reaction-diffusion, porous materials, nonlinear elasticity, glaciology (Diaz 1985, Pelisser 1975). Worldwide, Lions, Browder, Serrin, Necas, Mawhin, et al. worked in this field, while in Romania, Dinca in 1972 has a chapter dedicated to this domain. Among those who contributed in this field we count V.Barbu and I.Cioranescu. The results obtained by Dinca and his co-workers (J.Mawihn, P.Jebeleanu, P.Matei, J.Cringanu) refer to: i) the study of the qualitative properties for the duality mappings; ii) development of some topological methods (based on the properties of the Leray-Schauder degree) to obtain surjectivity; iii) use of abstract surjectivity results to the case of some remarkable differential operators which are duality mappings on suitable function spaces (see Dinca's CV).■ Multigrid methods may be viewed as domain decomposition methods when the problem is numerically solved by using the finite element spaces, with the advantage that the convergence rate is almost independent of the overlapping or mesh parameters. The large interest in such methods is underlined by the international conferences dedicated to these methods only which take place every year since 1987 while the number of papers published in journals is impressive. Although many problems in mechanics and engineering are modeled by inequalities, the number of papers in this direction is very small due to the difficulties in the convergence proofs. The main interest of the IMAR team did lie in the convergence study for the inequalities case (see Badea's CV) and we also mention collaborations with Universite Paris 13 and Ecole Polytechnique Federale de Lausanne. Convergence in case of non-variational inequalities, in comparison with variational inequalities, is much more difficult since they do not arise from the minimization of functionals and for many problems described by them, the existence and uniqueness of the solution can only be proved in some particular cases. We propose to study the convergence of multigrid methods for non-variational inequalities, frequently present in mechanical and engineering problems. We mention the results due to Kornhuber and Krause (Free University of Berlin); however, the theoretical proof of the convergence remains an open problem. ■ Many problems from continuum mechanics can be formulated as a minimum problems for a class of functionals of the form $v \rightarrow F\_f(v) = g(v) - <f,v>$, where $g:X \rightarrow (-\infty, \infty]$ is convex or not convex proper functional. For f in the dual of X the minimum problem for the functional F_f, is: find u of X such that $F\_f(u) \leq F\_f(v)$ for all v in X, is equivalent to the nonlinear equation: find u of X such that f belongs to the Gateaux subdifferential of g at u. In the last twenty years, were obtained a series of results in this direction, both at international and at national level, for example the works: Ionescu, Rosca and Sofonea (1985); Ionescu and Rosca (1990); Petryshyn (1992); Lu Chuang Zeng (1998); Hild, Ionescu, Lachand-Robert and Rosca (2002), (2003) and Zeqing Liu, Jeong Sheok Ume and

Shin Min Kang.■ The numerical solving of the nonlinear evolution equations represents a problem of large theoretical and applicative interest. As compared to the linear case, where there exist a rich literature and powerful solvers (ANSYS, FIDEUP, FEMLAB) the nonlinear problems are very difficult to be generalized. In this category we mention the quasilinear elliptic-parabolic equations. As the fields of applications of these equations are diffusion and infiltration processes, Stephan's problem, nonlinear heat propagation, nonlinear kinetic theory. We bring to the attention some of the most remarkable results in the domain: Alt and Luckhaus(1983), Carillo(1999) concerning the existence and uniqueness of the solutions, Eymard, Gutnic and Hilhorst(1988) concerning numerical approximation. Also we remark the papers of Marinoschi(2004), Marinoschi and Barbu(2003), Ion, Homentcovschi and Marinescu(2002) for water infiltration theory. Evolution equations defined by integral operators with strong nonlinear kernels as the Boltzmann models is another important domain. Here we remark the numerical approximation results obtained by Nambu(1980) and Grunfeld and Marinescu(1997). ■ The problem of realizing the self-propulsion from the motion of a helix foil on a helical surface has been widely studied (from the point of view of the numerical solution of the integral hyper-singular equations that one may encounter) because of the numerous applications in aeronautical and naval industries. In books like Anderson 1997, Wegener 1991, one may find important studies dedicated to this problem. From the mathematical point of view, there are the Euler equations which have to be solved in a domain whose boundary varies in time. On a national level, we have to mention the existence of a national school in this domain: in the framework of the problem of the small perturbations of the velocity and pressure for the subsonic flow (the linearized Euler equations), Homentcovschi 1975, deduced the integral equation of the lifting surface. Dragos (2003) and Dragos and Carabineanu (2002) have obtained quadrature formulas for the calculus of the finite part of the hyper-singular 2d integral. In the papers of Carabineanu (1999, 2003, 2004) the equation of the oscillatory lifting surface has been solved numerically and the self-propulsion was put into evidence (see CV - Carabineanu). ■ There are mechanical and electro-mechanical systems which can not be modelled in the frame of a lagrangean or hamiltonian formalism. In the last decade many papers have been devoted to a generalization, based on Dirac structure, of these formalisms and leading to implicit lagrangean or hamiltonian systems. The birkhoffian formalism is an alternative in the study of dynamic systems. In its coordinate independent form one comes to consider the formalism of 2 jets. Various implicit lagrangean and hamiltonian formulations have been considered in the study of electrical circuit theory; the advantages of a birkhoffian formalism have recently been presented by Ionescu and Scheurle (2004) for the dynamics of nonlinear LC circuits and electrical networks containing independent sources; Ionescu (2005) also obtained results in case of the dynamics of RLC nonlinear circuits (a post doc grant in the frame of the MASIE European net).(see Ionescu's CV). **Theme 3.** ■ The non-newtonian fluids are classes of fluids with many applications (from industry to biomechanics). El Kissi and all (1994, 1997, 1998), Saramito and Piau (1994), etc., putted into evidence (experimentally) various problems related to defects appearing in an extrusion process of an entangled polymer solution or melts. A relation between these defects and boundary conditions has been remarked (presence of some slipping regions). The proper mathematical models for these defects and implicitly the evaluation of "reasonable" solutions are, yet, an open problem. On the other hand, there are many models for such fluids (differential, rate type or integral models). A global review on this subject can be found in Larson (1988, 1999). The integral models which describe vary well the behaviour of nondilute solutions and polymer melts are difficult to be employed in real problems (Rajagopal and Winemann 1983, Tigoiu 1998). In the same time there are many papers focused on complex models of differential fluids. We mention here the second order fluid model (Coleman, Markovitz and Noll 1966, Dunn and Fosdick 1974, Fosdick and Rajagopal 1983, Tigoiu and Cipu 2000, 2005). Analytical solutions are not to be expected (when we use of these models in complex flows). We also underline here the important roll played by the knowledge of shear and elongation viscosities. Concerning this subject some partial results have been obtained during a collaboration with Laboratoire de Rheologie from Grenoble (see V.Tigoiu's CV). ■ There are other models being for long time utilized in the literature: second grade, Maxwell's and Oldroyd's models (Oldroyd, Dunn and Fosdick, Rajagopal and Srinivasa, Fetecau, Fetecau and Fetecau, etc). These models have been much studied and exact solutions (in various flows) have been obtained. Solutions for the problem of fluid motion over an infinite plate have been determined by: Bohme 1981, Rajagopal 1982, Fetecau and Zierep 2001 and Fetecau and Fetecau 2002. A study of the energetic equilibrium corresponding to different flows of Newtonian and non-newtonian fluids would be of a real interest. Recently, such a study, for the Rayleigh-Stokes problem for a Newtonian fluid was reported by Buhler and Zierep 2006. An important task is to extend this study to non-newtonian fluids (second grade, Maxwell, Oldroyd and possibly – Burgers fluids). (see Fetecau's CV). ■ There are different models used to describe the secondary oil recovery process: Hele-Shaw, Saffman-Taylor, Buckley-Leverett and the saturation model. In each case, studying the stability of the interface or of a basic solution for the saturation, is an important requirement. Some results in this direction have been obtained by : Bear – stability analysis of a basic solution, with an analogue of the Hele-Shaw model, but with different relative permeabilities for oil and the immiscible fluid; Yortsos and Hickernell – show that behind a certain value of the growth constant of the perturbation in the saturation model, the solution is stable independently of the "mobilities" of the 2 fluids; Barenblatt, Entov and Ryzhik – in the Buckley-Leverett model, the stability criterion is function of the mobilities of the 2 fluids. Moreover, there are results on stability improvement by Carasso and Pasa, Pasa, Daripa and Pasa, obtained in collaborations of IMAR with Universite de Saint-Etienne and Texas A&M University (see Pasa's CV). ■ The use of asymptotic methods for different practical problems is being extensively studied at the Universite de Saint-Etienne where prof. Panasenko's team has published a large number of papers on the subject. IMAR started a scientific collaboration with this team since the nineties and extended it after 2000 in the frame of the EURROMMAT Program (UE financed) and the CNCSIS (Romanian) Program ; significant results have thus been obtained in modelling the blood flow along an artery, starting with a simpler model and using consequently more and more realistic models. One has considered a flow domain with rigid boundaries and a periodic motion; all these results are published or are to be published in well-known international journals (see Stavre's CV). ■ Actually, the turbulence is an open problem of physics. Recently, Eringen (2005) proposed a theory of turbulence based on the dynamic micromorphic fluids. One requires a basic research of the qualitative properties of this new model. On the other hand, Eringen (2003) proposed a continuum theory for the mixture of a micropolar elastic solid and a viscous micropolar fluid. A large class of engineering materials like soils with grains, rocks, granular materials, sand and dirty fluids may be modeled, more realistic, with this theory. It is interesting to study the qualitative properties of the proposed model, which can prove that the model is well posed.

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3. OBJECTIVES

3.1 Categories of problems to be analyzed

3.1.1 Complex fundamental and applyed research activities

The aim of these activities is to perform a high research level for the development of mathematical modeling for the study of various materials as well as for the ellaboration of numerical and analytical technics and procedures for concrete problems. One intends also to increase the visibility of national research at international level.

3.1.2 National research orientation to European priorities in Material Sciences

The project aims to the adaption of the romanian reaserch area in materials study with a view to the thorough integration in the european area. We intend to identify the research dirrections, at partner level, and to follow the development and improvement of activities and human resources. Joint research activities an european priority directions will be achieved. Joint actual research subjects as well as the subjects demanding collaborations with other research centres will be identified.

3.2 Measurable objectives

3.2.1 Particular research directions. In the Project framework the following scientific objectives are considered:

Theme 1. ***Objective 1.1.*** The irreversible behavior of elasto-plastic materials with structural inhomogeneities of dislocation or twining structure types; ***Objective 1.2.*** Characterization of thermo-mechanic couppling in visco-plastic materials with phase changes with applications to new materials (multiphase steels, shape memory materials); ***Objective 1.3.*** The study of the functionally graded materials; ***Objective 1.4.*** Study of porous materials; ***Objective 1.5.*** Study of mixture materials.

Theme 2. ***Objective 2.1.*** Duality applications and boundary value problems in Orlicz-Sobolev spaces; ***Objective 2.2.*** Multigrid methods for non-variational inequalities. Applications to nonlinear problems in continuum mechanics or in engineering; ***Objective 2.3.*** Numerical and variational methods for solving boundary value problems in continuum mechanics; ***Objective 2.4.*** Approximation techniques and numerical algorithms for evolution operators. ***Objective 2.5.*** Methods of solving hypersingular integral equations in unsteady aerodynamics. ***Objective 2.6.*** Geometric study of Birkhoffian formalism with applications to dynamics of mechanical systems and electrical circuits.

Theme 3. ***Objective 3.1.*** Classes of Non-newtonian fluids of integral and differential types in complex motions (applications to nondilute polyner solutions and melts); ***Objective 3.2.*** Energetic equilibrium study for Rayleigh-Stokes' problem for some classes of non-newtonian fluids. ***Objective 3.3.*** The study of stability for Hele-Shaw, Buckley-Leverett and saturation models in secondary oil recovery; ***Objective 3.4.*** Asymptotic methods in mechanics and biomathematics; ***Objective 3.5.*** Study of fluid materials. New models of turbulence and solid-fluid mixture.

For achieving the above mentioned objectives we rely on the experience of the members of our team in mentioned domains (see CV and lists of scientific papers), existent connections with well known researchers all over the world and on already established links with major European research centers (see section 2). The research will be directed towards the achievement of both qualitative and quantitative results.

3.3 Concordance among proposed objectives and objectives and priorities of CEEX program. The present Project intends to turn to the best account the human resources, to colligate the activities of competent institutions in this domain and to help them to reach a coherent and efficient research team on order to upgrade the visibility of Romanian research in the field of mechanics. In the same time, other scopes of the Project are the improvement of researchers' skills to be involved in high level projects and the formation of young researchers able to develop research activities in the frame of European projects (FP7 for instance). The proposed objectives are part of general objectives of CEEX program, **TA-11** (they may be considered steps for future collaboration with teams developing research activities in the TA–4.2. Materials)

4. SCIENTIFIC AND TECHNICAL PRESENTATION OF THE PROJECT:

Our Project demands complex fundamental and applied research activities. All proposed problems are actual subjects open for international scientific community. The high complexity degree of proposed problems in our Project is given by actuality of the subjects starting from modeling of real phenomena and reaching to effective numerical results. We have to underline the interdisciplinary character of themes and approaches.

■ The research activity in the frame of **Theme 1** involves **CO, P1 and P2.** Young master and Ph.D. students are also involved. The team will collaborate with researchers from Universities of: Marseille, Metz, Poitiers, Salerno, Bologna, Texas A-M and with the Institute of Fundamental Technical Problems from Warsaw. **Objective 1.1.** The proposed research will be performed in two directions: *A)* To construct and to develop a sufficiently large mathematical model for second order theories, which allows the description structural inhomogeneities for continuum distributed dislocations and twining structures. We state: balance equations and virtual power principle; elastic type constitutive equations in relative to non-null torsion configurations for active forces (stresses and stress momentum) generalizing the second order deformation evolution (the metric and plastic connection); evolution laws for second order irreversible deformation; the concept of material symmetry for dislocations like inhomogeneities and multiple configurations materials. *B)* Formulation of: thermodynamic restrictions on homogeneous processes, for structural inhomogeneous materials; dissipation inequality for materials with uniformly distributed dislocations like inhomogeneities; deriving the constitutive restrictions and the roll of plastic spin and of the plastic torsion; the formulation of the dissipation inequality for twining structured materials and the analyze of the evolution equations roll into models with or without internal variables in the aim to describe twining like structures. **Objective 1.2.** The aim is to obtain a coherent description of thermo-viscoplastic, rate sensitive, phase transforming materials, by: formulation of the balance laws which take into account the heat exchange between the body and its surroundings; formulation of the rate-type constitutive laws by prescribing the evolution of the elastic-thermal-plastic and TRIP strains; investigating their compatibility with the second law of thermodynamics (with internal state variables) and determining the corresponding free energy; investigation of the free energy non-uniqueness, which leads to different heat propagation equations; formulation of initial and boundary value problems for the obtained PDE's; using energy and numerical methods in order to investigate the prediction of these models, with focus on the temperature distribution evolution during plastic deformation; comparison of numerical results with experimental data. Let us note that one will use experimental data obtained by our collaborators at the Universite de Metz and IPPT-PAN, Warsaw, which are leaders in temperature measurements by infrared emission. **Objective 1.3.** We are pointing out two subjects: *A)The study of the functionally graded materials in arch-like regions.* We consider a two-dimensional elastic state in the arch-like region (S): $a<r<b$, $0<\theta<\alpha$. The edges r =b, $\theta=0$ *and* $\theta=\alpha$ *are free of tractions, while the edge* r =a is subjected to equilibrated loads. For the functionally graded materials, the elastic characteristics depend continuous with respect to r or θ only. We intend to study the influence of the material inhomogeneity on the decay rate of the end effects of loads on the edge r =a at large distances to the loaded end. Such type of research is a real answer to the researches made by Flavin and Gleeson (2003), Chirita (2005), Chirita and D'Apice (2005). *B)Measures in terms of the displacement for constrained anisotropic cylinders.* We consider a prismatic cylinder made of an anisotropic elastic material with cross-sectional continuous inhomogeneity. The lateral surface is fixed and the boundary ends are subjected to given loads. We introduce some appropriate cross-sectional measures for the displacement and then we study the evolution with respect to the axial variable. The results established give a complete answer to the research started by Flavin, Knops and Payne (1989). **Objective 1.4.** The following two problems will be analyzed: *A) The study of the equations of motion of porous plates.* One consider the initial boundary value problems associated with the deformation of the plates of various types (plane plates or shells) made of porous media and then it will be studied the properties of solutions to these equations by establishing uniqueness, existence and continuous dependence results. To this end it will be used the semigroup theory of linear operators. There is the intention to include these results in a monograph related to the theory of porous plates of Mindlin type. *B) Deformation of the porous cylinder.* Consider a prismatic cylinder made of a porous elastic material and then we study an analytical solution of the boundary value problem in which the tractions are specified on the ends of cylinder and the lateral surface is free of tractions. By using the superposition method we obtain the stresses and equilibrated stresses near the boundary of cylinder. **Objective 1.5.**The following three questions are considered concerning this objective: *A) The study of the mathematical model of porous mixtures.* We will consider the initial boundary value problems associated with the linear theory of viscoelastic porous mixtures and then we will study the existence and temporal behaviour of solutions. To this end we will use some results concerning the theory of semigroups of linear operators and some identities of Brun-Lagrange type. *B) Mathematical model with microstretch for the micropolar theory of mixtures.* We intend to develop a theory of mixtures of porous media which allows for microrotations, as well as the possibility that each point can obey dilatations and contractions. *C) The study of the mathematical model with microstretch for the theory of micropolar mixtures.* We want to establish existence and continuous dependence results for the solutions of the corresponding initial boundary value problems.

■ The research activity in the frame of **Theme 2** involves **CO, P1 and P3.** Young master and Ph.D. students are also involved. The team will collaborate with researchers from Universities: Free of Berlin, Catholic of Louvain, Paris 6. **Objective 2.1.** The study of duality mappings is considered. The goal is to obtain qualitative properties, the surjectivity results, application to models of continuum mechanics, in the more general framework of Orlicz-Sobolev spaces. This approach has at least two major justifications. i) It is known that Orlicz spaces are a natural generalization of Lebesque spaces $L^p(\Omega)$, $1 < p < \infty$. The Orlicz-Sobolev spaces $W^m L_A(\Omega)$ are a generalization of Sobolev spaces $W^{m,p}(\Omega)$. It is natural to generalize the duality mapping from $W^{m,p}(\Omega)$ to Orlicz-Sobolev spaces $W^m L_A(\Omega)$ and to recover as many properties as possible . ii) In nonlinear elasticity were encountered the boundary problems for the operators which generalize the p-Laplacean (meaning that p is a continuous function $p:\overline{\Omega} \to \mathbf{R}$, with $p(x) > 1$, $\forall \quad x \in \overline{\Omega}$ -see Zhikov). To approach the boundary problem with p(x)-Laplacean a new functional framework is needed, namely the Orlicz-Sobolev spaces. **Objective 2.2.** The theoretical study of the multigrid methods for non-variational inequalities with the aim of proving the convergence of the methods and their error estimate; numerical applications in order to find effective solutions of some practical problems of real interest, such as contact problems with friction. We also intend starting a collaboration on this subject with prof. Kornhuber's team at the Free University of Berlin, where some results have been obtained by associating multigrid methods with a fixed point method. **Objective 2.3.** It is made an abstract study and a numerical modelisation of nonlinear equations of the form $f \in \partial g(u)$, for various functionals g, which are used in modelisation of problems coming from linear and nonlinear elasticity, viscoplasticity, and geomaterials' flow. Toward this goal the research will continue the works cited in section 2. The techniques used to prove the existence and uniqueness and to build generalized Sobolev solutions are : the method of energetic spaces and Friedrichs extensions for variational problems. To approximate solutions, the finite element method will be used. **Objective 2.4.** Our aim is to elaborate, to study and to set up the numerical schemes for a class of evolution equations and to perform numerical simulations. The approximation refers to the conservative quasilinear parabolic equations (PDE) and to the evolution equations generated by the nonlinear integral operators used in certain kinetic models. For PDE, the approximation of the solution is provided as the solution of some ordinary differential equations, *method of line*, obtained by the approximation of space differential operators. For this purpose we use the *finite volumes method*. We shall study the schemes as functions depending on the *triangulation* and *numerical flux*. In the case of kinetic models, in order to obtain efficient algorithms, we reformulate the equations in terms of positive and finite measures. For time integration we shall use the Euler method. To reduce the computational effort we shall use the Monte Carlo method to approximate the integral operators. For numerical simulation, we choose a problem from the infiltration theory: Richards' equation represents mathematical model for large class of diffusion phenomena modeled by singular equations (elliptic-parabolic). We shall study the numeric response of the model on different material functions, hydraulic behavior of different type soils, using our software packages as well as packages like FEMLAB.**Objective 2.5.** One assumes that the lifting surface *S(t)* moves on a helical surface. We consider that the helix foil (which has a rotation motion) oscillates around the medium position with the frequency λ. One obtains an integral equation for the lifting surface problem. Our aim is to give equivalent expressions for the lifting surface equation for a helical foil in order to put into evidence the kind of the singularities that we encounter inside the kernel. We split therefore the kernel of the equation into a sum of kernels in order to put into evidence certain types of singularities corresponding to certain kernels and to furnish adequate quadrature formulas. So, one obtains the discretised the equation and after finding the approximate solution, integrating numerically the jump of the pressure over the foil, one obtains the propulsion force. **Objective 2.6.** The birkhoffian systems describing an LC or RLC electrical circuit are conservative and dissipative, respectively. The functions considered in the above proofs may be used to construct Liapunov type functions and to prove, under certain conditions, stability results in these circuits. One studies consequently the existence of a variational structure associated to a birkhoffian system. One also looks for a theory of reducing the birkhoffian systems with symmetries. The same formalism will also be applied to other dynamical systems, such as mechanical systems with singularities.

■ The research activity in the frame of **Theme 3** involves **CO, P1 and P2.** Young master and Ph.D. students are also involved. The team will collaborate with researchers from Universities of: Savoie, Hamburg, Bayreuth, Saint-Etienne and Texas A-M. **Objective 3.1.** The problem of defects following an extrusion process is studied. The classes of integral constitutive equations (Green-Tobolsky and BKZ) and also two cinematic admissible fields (Binding and von Karman) will be employed. Some asymptotic methods will be utilized (in the sense of Tigoiu 1995, 2001, 2003). It is known (a partial result of Tigoiu 2002) that the elongation effects upon the defects of the extrudate can not be neglected. We will perform a study in order to compare shear and elongation effects. The above mentioned (experimental) results and remarks from El Kissi and all, Larson and Rajagopal and Srinivasa will be considered. Afterwards the linearised and/or asymptotic solution will be evaluated. Combining with the results from Ob.3.2. (relative to the boundary energetic equilibrium), which will be evaluated in terms of our models, we will revisit a result, obtained in 2004, concerning the slip-adherence condition. **Objective 3.2.** We extend, to non-newtonian fluids the results obtained by Buhler and Zierep for Newtonian fluids. First the three characteristic elements for the energetic equilibrium in the Rayleigh-Stokes problem for a second grade fluid will be determined. Exact and approximate (in the case when the constitutive modulus α_1 is small enough) solutions will be evaluated. The two Stokes' problems will be considered. After that approximate and exact solutions will be discussed for an Oldroyd-B fluid (for small relaxation times). We expect that the quantities corresponding to Maxwell's, second grade and to the Newtonian fluid respectively, will be recovered for particular values of relaxation times. As we can not have an exact solution in the case of Burgers' and generalized Burgers' fluids, we will discuss the second Stokes problem, only. **Objective 3.3.** One starts with the comparison of the obtained stability results by using Hele-Shaw, Buckley-Leverett models and the saturation model, with focus on the last (which is nonlinear) and neglecting absorption, adsorption and diffusion phenomena; one will prove that, under particular conditions, there exists a stable basic solution of the saturation model, independent of the wave number and mobilities. The particular case concerns the boundary conditions verified by saturation and some properties of the relative permeabilities. **Objective 3.4.** One resumes the asymptotic analysis of the interaction between a viscous fluid and an elastic boundary in case of non-homogeneous boundary conditions, when the deformations of the elastic structure are considered small enough to induce negligible deformations of the fluid domain. One investigates a non steady state viscous flow in a thin pipe with a viscoelastic boundary; the fluid motion is described by the Stokes equations and the boundary behaviour by the Sophie Germain equation. One approximates the solution of the fluid-structure interaction problem in a finite rectangle; one introduces the boundary layer correctors which allow the estimation of the error between the asymptotic solution and the macroscopic one. **Objective 3.5.** Firstly, one studies *Qualitative results concerning the theory of micropolar mixtures.* For the mathematical model of mixture of an elastic solid and a micropolar fluid, we intend to consider the linear theory in which we want to study the uniqueness and continuous dependence with respect to the given external data for the solutions of the corresponding initial boundary value problems. On the other hand, we will study: *The turbulence theory in the sense developed by Eringen.* It is expected to establish results which can prove that the proposed model is well defined.

5. PROJECT JUSTIFICATION:

5.1 Relevance of the project. The mathematical study of material behaviour is a very important and actual research domain worldwide. The subjects proposed in this project belong, according to sections 2,4 (see also the attached CV's), to the main trends of the international scientific community in the domain, through : the proposed problems, the mathematical and numerical techniques, the new mathematical models and methods used in approaching these problems; thus, we continue tradition and the special results of the Romanian school of continuum mechanics. A thorough qualitative study of the existent or proposed mathematical models is especially useful. For several such models, that fairly describe physical reality, the mathematical tools needed to analyse motion or deformation are still under construction. All the institutes involved in the present project as well as their research temas have the necessary experience and competence to obtain new scientific results in a large class of important research directions and may represent real scientific partners for future european collaboration in the frame of FP7 program. In this respect, the project is relevant as it leads to a higher level of scientific work in the above mentioned institutions, it develops and concentrates human and material resources, it involves youth in top research and it prepares the partner institutions for performance under european programs.