ELECTIVE COURSES

FOR THE ACADEMIC YEAR 2020-2021

MASTER PROGRAM

ADVANCED STUDIES IN MATHEMATICS

FIRST SEMESTER:

1. Analiză pe grupuri Lie
2. Teoria distribuțiilor și transformata Fourier
3. Aspecte geometrice în ecuații cu derivate parțiale și calcul variațional
4. Introducere în studiul ecuațiilor de evoluție
5. Algebre Lie
6. Analiză pe varietăți hermitiene

SECOND SEMESTER:

1. Analiză funcțională pentru lagrangianul Poincaré
2. Soluții slabe pentru ecuații de evoluție și legi de conservare
3. Elemente de teorie spectrală
4. Subvarietăți riemanniene
5. Algebre Hopf
6. Teoria analitică a numerelor

Fiecare student va face opțiuni, în ordinea preferințelor (pentru cele câte cinci cursuri pe care le va urma în fiecare semestru).

FISA UNITATII DE CURS

TITLU: Analiză pe grupuri Lie
DOMENIUL DE MASTER: MATEMATICĂ-ASM
STATUTUL: optional
NR.ORE/SAPTAMANA: 3 (Curs = 2; Seminar = 1)
SEMESTRUL: 3 / anul II de studiu
FORMA DE EXAMINARE: Examen
CREDITE: 5

OBIECTIVE:

Acest curs constituie o introducere în analiza armonică necomutativă, cu alte cuvinte studiul proprietatilor analitice ale funcţiilor definite pe grupuri necomuta-tive.

PROGRAMA:

Elemente de teorie Lie: grupuri Lie, algebre Lie, operatori diferentiali invarianti

Forme diferentiale invariante: actiuni de grupuri Lie, spatii omogene, masuri Haar, structuri simplectice

Spatii de functii pe grupuri compacte: operatori de convolutie, teoria spectrala operatorilor compacti, teoria caracterelor, teorema Peter-Weyl, aproximarea grupurilor compacte cu grupuri Lie

Reprezentari ale unor grupuri necompacte: grupuri Lie nilpotente, reprezentari induse, caracterele reprezentarilor unitare ireductibile

BIBLIOGRAFIE:

A.W. Knapp, Lie groups beyond an introduction." Second edition. Progress in Mathematics, 140. Birkhauser Boston, Inc., Boston, MA, 2002.

FISA UNITATII DE CURS

TITLU: Teoria distribuțiilor și transformata Fourier
DOMENIUL DE MASTER: MATEMATICĂ-ASM
STATUTUL: optional
NR.ORE/SAPTAMANA: 3 (Curs = 2; Seminar = 1)
SEMESTRUL: 3 / anul II de studiu
FORMA DE EXAMINARE: Examen
CREDITE: 5

OBIECTIVE:

Cursul isi propune sa ofere notiunile fundamentale din teoria distributiilor si transformata Fourier necesare oricarui matematician lucrand in domeniul Analizei Matematice. Cursul ar fi o continuare naturala cursului „Elemente de Analiza si Analiza Fourier" din anul I de Master (2019-2020).

PROGRAMA:

Functii test, spatii de functii test
Exemple de functii netede cu suport compact, avand suportul continut intr-o multime data.
Spatii de functii netede cu suport compact, spatiul functiilor Schwartz.

Definitia si proprietatile de baza ale distributiilor si ale distributiilor temperate.
Operatii cu distributii: derivare, inmultire cu functii netede, produs tensorial, convolutie.
Teorema lui Schwartz a nucleelor.
Transformarea Fourier pe distributii temperate
Definitie si proprietatile de baza.

Lema Riemann-Lebesque, teorema Plancherel, inegalitatea Hausdor -Young, teorema Paley-Wiener privind transformata Fourier a distributiilor cu suport compact.
Solutii elementare.

BIBLIOGRAFIE:

L. Hormander, The analysis of linear partial di erential operators. I. Distribution theory and Fourier analysis. Second edition. Grundlehren der Mathematischen Wissenschaften 256. Springer-Verlag, Berlin, 1990.

V. Iftimie, Ecuatii cu derivate partiale (Anul III Matematica), Editura Universitatii Bucuresti, 1989.

FISA UNITATII DE CURS

TITLU: Aspecte geometrice în ecuații cu derivate parțiale și calcul variațional (**GEOMETRIC ASPECTS IN PARTIAL DIFFERENTIAL EQUATIONS AND CALCULUS OF VARIATIONS)**

DOMENIUL DE MASTER: MATEMATICĂ-ASM
STATUTUL: optional
NR.ORE/SAPTAMANA: 3 (Curs = 2; Seminar = 1)
SEMESTRUL: 3 / anul II de studiu
FORMA DE EXAMINARE: Examen
CREDITE: 5

OBIECTIVE:

The purpose of this course oriented to master students (2nd year, first semester) is to present some interesting well-known results in partial differential equations (PDE) and calculus of vari-ations in which the geometry of the domain plays a crucial role. Particularly, this is in strong connection with differential geometry tools usually taught in the undergraduate courses and geometric measure theory tools.
The course is divided in three main parts: 1) The first part is focused on overdetermined problems in which the additional boundary constrains force the domain to have a special geometry (ball, interior of an ellipse, etc.) and the solutions to have specific symmetric properties; 2) The second part deals with symmetric re-arrangements and isoperimetric-type inequalities to subsequently determine the asymptotic behaviour of the first eigenvalue of the p-Laplace operator as p tends to 1; 3) The third part gravitates around the Sobolev inequality and its applications to nonlinear PDE.

PROGRAMA:

1. **Second order equations with overdetermined boundary conditions**

Poisson (Serrin) problem.
Monge-Ampère problem.
Other examples (liniar or nonlinear).
Main tools: Pohozaev-type identities, maximun principles, moving plane method, properties of matrices.

2. **Symmetric rearrangements and minimization problems in calculus of variations**

Decreasing rearrangements. Invariance of the Lp-norms. Hardy-Littlewood inequality.
Schwarz symmetrization. Isoperimetric inequality in any dimension. Federer co-aria formula. Pólya-Szegö theorem.
The Cheeger constant : definition and explicit deter-mination in special domains (balls).
Dirichlet p-Laplace operator. Variational characterization of the first eigenvalue. Faber-Krahn type inequality.
Application.

Main tools: fundamental results in geometric measure the-ory, approximation arguments.

3. **Sobolev inequality and applications**

Best Sobolev constant. Talenti's explicit formula for $p > 1$.
Equivalence between the isoperimetric inequality and thebest Sobolev constant in the case $p = 1$.
Existence of positive solutions to some nonlinear PDEs with critical Sobolev exponent.
Main tools: Schwarz symmetrization and dimension re-duction, ordinary differential equations, geometric measure theory tools, basic functional analysis tools.

BIBLIOGRAFIE:

[1]2B. Brandolini, C. Nitsch, P. Salani, C. Trombetti, *Serrin-type overdetermined problems: an alternative proof*. Arch. Ration. Mech. Anal. 190 (2008), no. 2, 267–280.
[2] H. Brezis, L. Nirenberg, *Positive solutions of nonlinear elliptic equations involving critical Sobolev exponents*. Comm. Pure Appl. Math. 36 (1983), no. 4, 437–477.
[3] J. Cheeger, *A lower bound for the smallest eigenvalue of the Laplacian*, in: Problems in Analysis, A Symposium in Honor of Salomon Bochner, Ed.: R.C.Gunning, Princeton Univ. Press (1970) 195–199.
[4] C. Enache, private communication, www.aus.edu/faculty/dr-cristian-enache.
[5] B. Kawohl, *Rearrangements and convexity of level sets in PDE*. Lecture Notes in Mathematics, 1150. Springer-Verlag, Berlin, 1985.
[6] B. Kawohl, V. Fridman, *Isoperimetric estimates for the first eigenvalue of the* p-*Laplace operator and the Cheeger constant.* Comment. Math. Univ. Carolin. 44 (2003), no. 4, 659–667.
[7] S. Kesavan, *Symmetrization & applications*. Series in Analysis, 3. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2006.
[8] G. Talenti, *Best constant in Sobolev inequality*. Ann. Mat. Pura Appl. (4) 110 (1976), 353–372.
[9] J. Serrin, *A symmetry problem in potential theory*, Arch. Rational Mech. Anal. 43 (1971), 304–318.

FISA UNITATII DE CURS

TITLU: Introducere în studiul ecuațiilor de evoluție

DOMENIUL DE MASTER: MATEMATICĂ- ASM
STATUTUL: optional
NR.ORE/SAPTAMANA: 3 (Curs = 2; Seminar = 1)
SEMESTRUL: 3 / anul II de studiu
FORMA DE EXAMINARE: Examen
CREDITE: 5

OBIECTIVE:

Cursul prezintă câteva tehnici de bază pentru a obține bine-punerea unor ecuații de evoluție: căldură, convecție-difuzie, unde/ Klein-Gordon, Schrödinger, etc...
Cursul prezintă teoria clasică pentru acest tip de ecuații urmând textul Cazenave-Haraux, An introduction to semilinear evolution equa-tions, [2]. Vom incepe cu câteva elemente de spații de funcții și opera-tori nemărginiți. Urmează apoi abordarea problemelor liniare omogene si neomogene folosind elemente de semigrupuri. Se intră apoi în cazurile particulare principale: căldură, unde/Klein-Gordon si Schrödinger, unde se abordează probleme neliniare. Toate aceste elemente sunt acoperite in primele sapte capitole din textul propus.

PROGRAMA

Preliminarii de spatii si analiză functională, functii cu valori vectoriale, spatiile $L^p((0, T), X)$

Operatori nemărginiti în spatii Banach, operatori maximali disipativi, operatori nemărginiti în spatii Hilbert, exemple din teoria ecuațiilor cu derivate partiale

Teorema Hille - Yosida și aplicații, extrapolare si soluții slabe, semigrupuri de contractii si generatori, exemple.

Probleme neomogene si probleme semiliniare abstracte

Cazuri particulare fundamentale
căldură $u_t = \Delta u + f(u)$
Klein-Gordon $u_{tt} - \Delta u + mu = f(u)$
Schrödinger, $iu_t + \Delta u = f(u)$

BIBLIOGRAFIE:

Haim Brezis, Functional analysis, Sobolev spaces and partial differential equa-tions, Universitext, Springer, New York, 2011. MR 2759829

T. Cazenave and A. Haraux, An introduction to semilinear evolution equations. Transl. by Yvan Martel. Revised ed., Oxford Lecture Series in Mathematics and its Applications. 13. Oxford: Clarendon Press. xiv, 1998.

FIȘA UNITĂȚII DE CURS

TITLU: Lie Algebras

DOMENIUL DE MASTER: MATEMATICĂ- ASM
STATUTUL: optional
NR.ORE/SAPTAMANA: 3 (Curs = 2; Seminar = 1)
SEMESTRUL: 3 / anul II de studiu
FORMA DE EXAMINARE: Examen
CREDITE: 5

OBIECTIVE:

1. Understanding of basic concepts of the theory Lie algebras.
2. Knowledge of the main properties of Lie algebras, especially nilpotence, solvability, and semisimplicity.
3. Knowledge of Lie algebra representations.
4. Knowledge and understanding of the Killing-Cartan classification of the finite dimensional simple Lie algebras

PROGRAMA:

1. The basic notions of Lie Algebras Theory.
2. Modules and representations. The classification of representations of **sl**(2, **C**).
3. Nilpotent Lie algebras, Engel Theorem.
4. Solvable Lie algebras, Lie Theorem.
5. Semisimple Lie algebras, the Killing form, Cartan's criteria.
6. Irreducible representations, Weyl Theorem.
7. Root systems, the root space decomposition.
8. The classification of semisimple Lie algebras (presentation of the main results, without proofs).

BIBLIOGRAFIE:

1. K. Erdmann, M. J. Wildon. *Introduction to Lie Algebras*. Springer (2006)
2. J.E. Humphreys. *Introduction to Lie Algebras and Representation Theory*, Graduate Texts în Mathematics. Springer (1997)
3. N. Jacobson. Lie algebras, Interscience Publishers (1961)

Ian M. Musson, *Lie Superalgebras and Enveloping Algebras*, American Mathematical Society, Providence, RI, 2012.

FIȘA UNITĂȚII DE CURS

TITLU: Analysis on Complex Manifolds

DOMENIUL DE MASTER: MATEMATICĂ- ASM
STATUTUL: optional
NR.ORE/SAPTAMANA: 3 (Curs = 2; Seminar = 1)
SEMESTRUL: 3 / anul II de studiu
FORMA DE EXAMINARE: Examen
CREDITE: 5

OBIECTIVE:

Knowledge and understanding of the interplay between analysis, geometry, algebra and topology, with emphasis on differential analysis.
Understanding the influence of analyticity on geometric properties.
Ability to use complex analytic tools in geometry problems.

PROGRAMA

1. Complex manifolds.
2. Complex and holomorphic vector bundles.
3. Hermitian manifolds.
4. The Chern connection.
5. Chern-Weil theory.
6. Kaehler manifolds.
7. Hermite-Einstein and Kaehler-Einstein metrics.
8. Elliptic Operator Theory.
9. Introduction in Hodge theory.
10. Submanifolds.

BIBLIOGRAFIE:

1. R. O. Wells, Differential Analysis on Complex Manifolds. Springer, 1980.
2. P. Griffith, J. Harris, Principles of algebraic geometry, Wiley, 1994.
3. J. P. Demailly, Complex Analytic and Differential Geometry, https://www-fourier.ujf-grenoble.fr/~demailly/manuscripts/agbook.pdf
4. D. Huybrechts, Complex Geometry, Springer, 2004.

FISA UNITATII DE CURS

TITLU: Analiză funcțională pentru lagrangianul Poincaré
DOMENIUL DE MASTER: MATEMATICĂ-ASM
STATUTUL: optional
NR.ORE/SAPTAMANA: 3 (Curs = 2; Seminar = 1)
SEMESTRUL: 4 / anul II de studiu
FORMA DE EXAMINARE: Examen
CREDITE: 5

OBIECTIVE:

In acest curs vom prezenta unele metode de analiza functionala in studiul lagrangianului Poincare. Dupa cum a remarcat Poincare - ecuatia fortei Lorentz (nucleul teoriei speciale a relativitatii) este ecuatia Euler - Lagrange asociata lagrangianului Poincare. Vom demonstra riguros cu tehnici de analiza functionala ca rationamentul lui Poincare are corespondent modern. Utilizand aceasta abordare moderna a lagrangianului Poincare, vom arata unele rezultate de existenta a solutiilor periodice pentru ecuatia fortei Lorentz. Aceste solutii se vor obtine ca puncte de minim ori min-max pentru lagrangianul Poincare. In functie de timp si de nivelul clasei, vom incerca sa aratam ca simetria lagrangianului Poincare (adica simetria campului electro-magnetic) genereaza multiplicitatea solutiilor pentru ecuatia fortei Lorentz adica vom prezenta o teorie Lusternick-Schnirelman pentru lagrangianul Poincare.

PROGRAMA:

Notiuni introductive de teoria masurii si analiza functionala (teorema

Leibniz-Newton in contextul integralei Lebesgue, functii Lipschitz, teorema Banach-Alaoglu in spatii Hilbert si mini-mizarea functionalelor slab inferior semicontinue.
Functionala Poincare ca suma dintre o functionala convexa si una neteda. Puncte critice in sens Szulkin.

Principiul actiunii minime pentru lagrangianul Poincare (adica o teorema de tip Banach-Mazur).

O teorema de min-max si aplicatii la existenta solutiilor periodice pentru ecuatia fortei Lorentz utilizand lagrangianul Poincare.

Puncte critice multiple pentru functionale impare de tipul lagrangianului Poincare cu camp electro-magnetic simetric.

BIBLIOGRAFIE:

D. Arcoya, C. Bereanu, P. Torres, Critical point theory for the Lorentz force equation, Arch. Rational Mech. Anal., 232 (2019), 1685-1724.

D. Arcoya, C. Bereanu, P. Torres, Lusternik-Schnirelman theory for the action integral of the Lorentz force equation, Calc. Var. PDE, (2020) 59-50.

FISA UNITATII DE CURS

TITLU: Soluții slabe pentru ecuații de evoluție și legi de conservare
WEAK SOLUTIONS OF EVOLUTION EQUATIONS AND CONSERVATION LAWS

DOMENIUL DE MASTER: MATEMATICĂ-ASM
STATUTUL: optional
NR.ORE/SAPTAMANA: 3 (Curs = 2; Seminar = 1)
SEMESTRUL: 4 / anul II de studiu
FORMA DE EXAMINARE: Examen
CREDITE: 5

OBIECTIVE:

In the first part we continue the PDE course taught for 3rd year undergraduate students at FMI-UB. This proposal is complementary to the course Evolution Equations. We introduce the notion of weak solutions of homogeneous/nonhomogeneous parabolic and hyper-bolic linear equations and study techniques to prove the existence and uniqueness of such weak solutions.

In the second part of the course we study rst order nonlinear equations, being focused on conservation laws for which we analyze various properties of weak solutions such as existence/uniqueness and asymptotic behavior

PROGRAMA:

Second order parabolic equations. De nition and existence of weak solutions. Galerkin approximations. Energy methods, uniqueness and regularity of weak solutions. Maximum princi-ple and Harnack inequalities.

Second order hyperbolic equations. De nition of weak solu-tions, existence and uniqueness. Energy estimates, regularity and nite speed of propagation.

First order nonlinear equations. Characteristic curves. Conser-vation laws. Integral solutions. Rankine-Hugoniot condition. Shock waves. The entropy condition/Lax-Oleinik formula. The entropy solution. Riemann problem. Asymptotic behavior of the entropy solution.

BIBLIOGRAFIE:

H. Brezis, Functional analysis, Sobolev spaces and partial differential equations. Universitext. Springer, New York, 2011.

L. C. Evans, Partial differential equations, Graduate Studies in Mathematics, 19, Second Edittion, American Mathematical Society, Providence, RI, 2010.

F. John, Partial di erential equations. Applied Mathematical Sciences, Springer-Verlag, New York, 1991.

J. Rauch, Partial differential equations. Graduate Texts in Mathematics,128. Springer-Verlag, New York, 1991.

FISA UNITATII DE CURS

TITLU: Elemente de teorie spectrală

DOMENIUL DE MASTER: MATEMATICĂ-ASM
STATUTUL: optional
NR.ORE/SAPTAMANA: 3 (Curs = 2; Seminar = 1)
SEMESTRUL: 4 / anul II de studiu
FORMA DE EXAMINARE: Examen
CREDITE: 5

OBIECTIVE:

Cursul se bazeaz pe cursul de analiz functionala din anul III. Vor prezentate notiunile importante de teorie spectral a operatorilor liniari in spatii Banach si Hilbert, urmarindu-se aplicatiile in probleme de ecuatii.

PROGRAMA:

Elemente de teoria operatorilor; spectru, rezolventa.
Teoria spectral a operatorilor compacti.
Teoria spectral a operatorilor autoadjuncti si normali in spatii Hilbert.
Operatori nemarginiti in spatii Hilbert.
Semigrupuri de operatori.

BIBLIOGRAFIE:

M. Reed, B. Simon: Methods of Modern Mathematical Physics: vol.1: Functional Analysis, Academic Press, 1980.

FISA UNITATII DE CURS

TITLU: Subvarietăți riemanniene
Submanifolds of Riemannian Manifolds

DOMENIUL DE MASTER: MATEMATICĂ- ASM
STATUTUL: optional
NR.ORE/SAPTAMANA: 3 (Curs = 2; Seminar = 1)
SEMESTRUL: 4/ anul II de studiu
FORMA DE EXAMINARE: Examen
CREDITE: 5

OBIECTIVE:

Knowledge and understanding of the relation between the geometry of submanifolds and the geometry of the ambient space.
Understanding how Riemannian invariants obstruct the existence of certain type of submanifolds.

PROGRAMA

1. Submanifolds in Riemannian manifolds. Gauss and Weingarten formulae. Gauss, Codazzi and Ricci equations.
2. Totally geodesic submanifolds, minimal submanifolds, totally umbilical submanifolds.
3. Invariants of Riemannian manifolds: scalar curvature, Ricci curvature, Chen invariants.
4. Optimal geometric inequalities for Riemannian invariants of submanifolds in constant curvature spaces.
5. Applications: obstructions to the existence of minimal submanifolds, classification theorems of ideal submanifolds.
6. Special classes of submanifolds in Hermitian manifolds: complex submanifolds, Lagrangian submanifolds, slant submanifolds, CR-submanifolds..

BIBLIOGRAFIE:

1. D. Blair, *Riemannian Geometry of Contact and Symplectic Manifolds*, Birkhauser, 2010.
2. B.Y. Chen, *Geometry of Submanifolds,* M. Dekker, 1973.
3. B.Y. Chen, *Pseudo-Riemannian Geometry, Delta Invariants and Applications,* World Scientific, 2011.
4. S. Kobayashi, K. Nomizu, *Foundations of Differential* Geometry, vol. II, Wiley-Interscience, 1969.
5. I. Mihai, *Geometria Subvarietăților în Varietăți Complexe,* Editura Universității din București, 2002.
6. I. Mihai, A. Mihai, V. Ghișoiu, *Culegere de Probleme de Geometrie Diferențială,* Editura Universității din București, 2012.

FIȘA UNITĂȚII DE CURS

TITLU: Algebre Hopf
Hopf Algebras

DOMENIUL DE MASTER: MATEMATICĂ- ASM
STATUTUL: optional
NR.ORE/SAPTAMANA: 3 (Curs = 2; Seminar = 1)
SEMESTRUL: 4 / anul II de studiu
FORMA DE EXAMINARE: Examen
CREDITE: 5

OBIECTIVE:

1. Knowledge and understanding of the concepts of algebra and its dual, coalgebra.
2. Understanding the relevance of representations.
3. Understanding the role of algebraic methods in geometry and analysis.

PROGRAMA:

1. Algebras and coalgebras.
2. Modules and comodules.
3. Bialgebras and Hopf algebras.
4. Hopf modules.
5. Integrals for Hopf algebras.
6. Semisimple Hopf algebras.
7. Actions and coactions of Hopf algebras.

BIBLIOGRAFIE:

1. M. Sweedler, *Hopf Algebras*, Benjamin, New-York, 1969.
2. E. Abe, *Hopf Algebras*, Cambridge Univ. Press., 1977.
3. S. Dăscălescu, C. Năstăsescu, Ş. Raianu, *Hopf algebras: an introduction*, Marcel Dekker, 2000
4. D. E. Radford, *Hopf Algebras*, World Scientific, 2012
5. Tomasz Brzezinski, Robert Wisbauer, *Corings and Comodules*, Series: London Mathematical Society Lecture Note Series (No. 309), Cambridge University Press, 2003.
6. Hazewinkel, Michiel; Gubareni, Nadiya; Kirichenko, V. V. *Algebras, rings and modules*. *Lie algebras and Hopf algebras*. Mathematical Surveys and Monographs, 168. American Mathematical Society, Providence, RI, 2010.

FIȘA UNITĂȚII DE CURS

TITLU: Teoria analitică a numerelor
Analytic Methods in Number Theory

DOMENIUL DE MASTER: MATEMATICĂ- ASM
STATUTUL: optional
NR.ORE/SAPTAMANA: 3 (Curs = 2; Seminar = 1)
SEMESTRUL: 4 / anul II de studiu
FORMA DE EXAMINARE: Examen
CREDITE: 5

OBIECTIVE:

1. Knowledge and understanding of the interplay between analysis, algebra and number theory.
2. Understanding the influence of analyticity on number theory problems.
3. Ability to use analytic tools in number theory problems.

PROGRAMA

1. Prime Number Theorem.
2. The Riemann zeta function.
3. Zero-free region for zeta function.
4. Riemann Hypothesis.
5. Prime numbers in arithmetical sequences. Dirichlet's theorem.
6. Minkowski's theorem of the convex body.
7. Schnirelmann density.
8. Circle method.
9. Analytical formulas for the class numbers

BIBLIOGRAFIE:

1. Gica A., Panaitopol L., *Probleme celebre de teoria numerelor*, Editura Universităţii Bucureşti, 1998.
2. Jean-Marie De Koninck, Florian Luca, *Analytic Number Theory: Exploring the Anatomy of Integers* (Graduate Studies in Mathematics), AMS, 2012.
3. Henryk Iwaniec and Emmanuel Kowalski, *Analytic Number Theory*, American Mathematical Society, Providence, RI, 2004.