University of Bucharest

Faculty of Mathematics and Computer Science

Summary of Ph.D Thesis

Conformal Geometry on Hermitian and Symplectic Manifolds

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Overview of the thesis and outline of the original results

This thesis aims at studying the geometry of manifolds endowed with a locally conformally symplectic (LCS)/locally conformally Kähler (LCK) structure and it is based on the results in [Ot1], [Ot2] and [AOT].

By LCK structure on a complex manifold (M, J) we mean a Hermitian metric g, whose fundamental two-form ω satisfies:

$$d\omega = \theta \wedge \omega \tag{1}$$

for a closed one-form θ, called the Lee form of the metric.

The non-metric version of this definition, namely, a non-degenerate two form satisfying (1) is the definition of LCS. Condition (1), which we shall often refer to as the *LCS condition* rewrites as $d_\theta\omega = 0$, where $d_\theta := d - \theta \wedge \cdot$.

The operator d_θ defines a cohomology, which makes perfectly sense on every smooth manifolds, with respect to any closed one-form θ, but for LCS/LCK it is very natural to consider it with respect to the Lee form.

We briefly present the way the thesis is organized and we describe the main results which have a twofold goal. The first is to generalize constructions from Kähler and symplectic geometry to the LCK/LCS setting, with the purpose of finding new exemples of LCS manifolds. The second goal is to study the twisted cohomological aspects and what kind of information about the LCK structures lies in this type of cohomology, our focus being on complex surfaces.

The first chapter reviews the main properties of LCK/LCS, already known in the literature, such as equivalent definitions, examples, the special subclasses of Vaisman and LCK manifolds with potential, parallel LCK vs LCS etc. The results presented in this introductory chapter and the preliminaries of each chapter are the only prerequisites to make the results of the paper comprehensible.

Chapter 2. Locally conformally symplectic bundles. Main results

In the second chapter, we define *locally conformally symplectic bundles*, an analogue of *symplectic bundles*, a notion introduced by Guillemin, Sternberg, Weinstein *et al.* (see [GLSW], [St], [We]). A very detailed exposition of symplectic bundles can also be found in [MS]. A symplectic bundle is a locally trivial fibration, with symplectic fiber, admitting a trivialization with transition maps acting as symplectomorphisms with respect to the symplectic form of the fiber. The reason behind considering this notion was trying to find new examples of symplectic manifolds among these objects. This motivates us to extend the search of new examples of LCS manifolds among the total space of fiber bundles whose fibre already has an LCS structure invariant under the action of the transition maps. This is what we call *LCS bundle* or *LCS fibration*.

A well known construction of Sternberg, called *the coupling form*, gives sufficient conditions when a symplectic fiber bundle is itself symplectic, by making use of Hamiltonian actions. The idea is to join a symplectic manifold on which a Lie group G acts Hamiltonianly and a principal G-bundle, admitting a so-called *fat* connection, a technical requirement, which means the principal bundle has *a lot of curvature*, quite the opposite of flat bundle. For more details on fat connections, one can check [We]. We state the result of Sternberg and Weinstein:

Theorem 0.1: ([St, We]) Let (F,ω) be a symplectic manifold with a Hamiltonian action of a Lie group G on F. If $\mu : F \to g^*$ is the momentum map, then any connection on a G-principal bundle P which is fat at all the points in $\mu(F)$ induces a symplectic form on $P \times_G F$.

The extension to the LCS setting requires a Hamiltonian group action with respect to an LCS structure. Twisted Hamiltonian actions and the corresponding reduction procedure have been studied by I. Vaisman, S. Haller, T. Rybicki, R. Gini, L. Ornea, M. Parton, and more recently by F. Madani, A. Moroianu and M. Pilca and we use them in order to prove the main result of Chapter 2:

Theorem 0.2: ([Ot1]) Let (F,ω,θ) be a locally conformally symplectic manifold not globally conformally symplectic and let G be a Lie group acting on F by diffeomorphisms preserving ω (and hence θ). If the action of G is twisted Hamiltonian and if $\mu : F \to g^*$ is a momentum map of the action, then any connection on a G - principal bundle P which is fat at $\mathrm{Im}\mu$ induces an LCS structure on $P \times_G F$.

We study several examples of twisted Hamiltonian actions and also compatibility properties with respect to the reduction process. We prove the next two results:

Theorem 0.3: ([Ot1]) Let $\pi : \overline{M} \to M$ be the minimal covering of the LCS manifold (M,ω,θ) and let Ω be the symplectic form of $\overline{M}$. Let G be a Lie group acting on M by preserving ω. Then G acts twisted Hamiltonianly on M if and only $\tilde{G_0}$ acts Hamiltonian on $\overline{M}$, where $\tilde{G_0}$ is the connected component of the identity of the universal covering of G.

Theorem 0.4: ([Ot1]) Let G be an abelian Lie group acting twisted Hamiltonian on the LCS manifold (F,ω,θ) and P a principal G-bundle with a fat connection at $\mathrm{Im}\,\mu$. Then G acts twisted Hamiltonianly with respect to the coupling form Ω and assuming all conditions for reduction at 0 are met, the relation between the two reduced manifolds is:

$$\tilde{\mu}^{-1}(0)/G \simeq P/G \times \mu^{-1}(0)/G.$$

Chapter 3. Morse-Novikov cohomology. Main results

Chapter 3 and 4 are both concerned with studying the twisted cohomological properties of compact complex LCK manifolds, however in the third chapter the emphasis is on the LCK complex surfaces, and in the last one we concentrate on LCK solvmanifolds, particularly on Oeljeklaus-Toma manifolds. Beside the motivation of the natural setting that LCK manifolds provide for the twisted cohomology, we study it also for understanding better the nature of their LCK metrics.

The starting point for the results in Chapter 3 was [AD], where the authors characterize the possible Lee forms for LCK structures, and more generally for LCS structure which tame the complex structure, for the following complex surfaces: Inoue surfaces $\mathscr{S}^{\pm}$, Kato surfaces, Hopf surfaces. In [AD], it is considered the subset of H^1_{dR} consisting of the possible Lee forms of LCK metrics, respectively of tamed LCS structure:

$$\mathscr{C}(X) = \{[\theta] \in H^1_{dR}(M) \mid \text{there exists } \omega \in \Omega^{1,1}(X), \omega > 0, d_\theta \omega = 0\}$$

$$\mathscr{T}(X) = \{[\theta] \in H^1_{dR}(M) \mid \text{there exists } \omega \in \Omega^{2}(X), \omega^{1,1} > 0, d_\theta \omega = 0\}$$

The result in [AD] that motivates our interest in the twisted cohomology of LCK complex surfaces is:

Theorem 0.5: [AD] Let S be a complex surface with the minimal model S_0 (i.e. S_0 is not the blow-up of another complex surface). Then:

- If S_0 is a Hopf surface, then $\mathscr{C}(S_0) = \mathscr{T}(S_0) = (-\infty, 0)$.
- If S is an Inoue surface of type $\mathscr{S}^+_{N,p,q,r,z}$ with $z \in \mathbb{C} \setminus \mathbb{R}$, then $\mathscr{C}(S) = \emptyset$ and $\mathscr{T}(S) = \{a_0\}$.
- If S is an Inoue surface of type $\mathscr{S}^+_{N,p,q,r,z}$ with $z \in \mathbb{R}$, then $\mathscr{C}(S_0) = \mathscr{T}(S_0) = \{a_0\}$.

Here $H^1_{dR}(S) \simeq \mathbb{R}$ is identified with the oriented line $(-\infty, \infty)$ and $a_0 \in H^1_{dR}(S)$ is the class whose associated holomorphic line bundle is the anti-canonical bundle $\mathscr{K}^*_S$.

The main result in Chapter 3 is :

Theorem 0.6: ([Ot2]) Let $\mathscr{S}$ be an Inoue surface. Then:

- If $\mathscr{S} = \mathscr{S}^0$, then $H^i_\theta(\mathscr{S}) = 0$, for $i = 0,1,4$, $H^2_\theta(\mathscr{S}) \simeq H^3_\theta(\mathscr{S}) \simeq \mathbb{R}$. By Poincaré duality, $H^i_{-\theta}(\mathscr{S}) = 0$, for $i = 0,3,4$, $H^1_{-\theta}(\mathscr{S}) \simeq H^2_{-\theta}(\mathscr{S}) \simeq \mathbb{R}$ and $H^i_{t\theta}(\mathscr{S}) = 0$, for all $i \geqslant 0$ and $t \neq \pm 1$.
- If $\mathscr{S} = \mathscr{S}^+_{A,p,q,r,z}$ with $z \in \mathbb{R}$, then $H^i_\theta(\mathscr{S}) = 0$, for $i = 0,4$, $H^1_\theta(\mathscr{S}) \simeq H^3_\theta(\mathscr{S}) \simeq \mathbb{R}$, $H^2_\theta(\mathscr{S}) \simeq \mathbb{R}^2$. By Poincaré duality, $H^i_{-\theta}(\mathscr{S}) \simeq H^i_\theta(\mathscr{S})$, and $H^i_{t\theta}(\mathscr{S}) = 0$, for all $i \geqslant 0$ and $t \neq \pm 1$.
- If $\mathscr{S} = \mathscr{S}^+_{A,p,q,r,z}$ with $z \in \mathbb{C} \setminus \mathbb{R}$, then, since $\mathscr{S}$ is diffeomorphic to some $\mathscr{S}^+_{A,p,q,r,z}$, with $z \in \mathbb{R}$, we denote by θ_1 be the image of θ via this diffeomorphism. The same result as in the previous case holds for $\mathscr{S}$ with θ_1 instead of θ.
- If $\mathscr{S} = \mathscr{S}^-$, $H^i_\theta(\mathscr{S}) = 0$, for $i = 0,1,4$, $H^2_\theta(\mathscr{S}) \simeq H^3_\theta(\mathscr{S}) \simeq \mathbb{R}$. By Poincaré duality, $H^i_{-\theta}(\mathscr{S}) = 0$, for $i = 0,3,4$, $H^1_{-\theta}(\mathscr{S}) \simeq H^2_{-\theta}(\mathscr{S}) \simeq \mathbb{R}$ and $H^i_{t\theta}(\mathscr{S}) = 0$, for all $i \geqslant 0$ and $t \neq \pm 1$,

where θ denotes in each case the Lee form of the LCK metric described by Tricerri in [Tr].

First notice that twisted cohomology distinguishes between the surfaces $\mathscr{S}^+$ and $\mathscr{S}^-$, although usual de Rham cohomology does not and $\mathscr{S}^+$ is a cover with two sheets of $\mathscr{S}^-$.

This helps us to complete the list of Apostolov and Dloussky with the Inoue surface $\mathscr{S}^0$ and state:

Corollary 0.7: ([Ot2]) $\mathscr{C}(\mathscr{S}^0) = \mathscr{T}(\mathscr{S}^0) = \{[\theta]\}$.

We further characterize the LCK metrics on $\mathscr{S}^0$:

Corollary 0.8: ([Ot2]) The fundamental form Ω of any LCK metric on $\mathscr{S}^0$ is of the form $\omega + d_\theta\eta$ for some closed one-form η.

In the corollaries above, (ω, θ) denotes the LCK structure found by Tricerri on $\mathscr{S}^0$ in [Tr].

As a by-product, we obtain that the complex line bundle associated to θ is the anticanonical bundle $\mathscr{K}^*_S$ and that Oeljeklaus-Toma (OT) manifolds do not admit LCK metrics which are d_θ-exact. The main ingredient in computing the Morse-Novikov cohomology of $\mathscr{S}^0$ and $\mathscr{S}^\pm$ is the twisted Mayer Vietoris sequence, detailed in [HR1]. Nevertheless, we present an alternative method using spectral sequences, which inspires a new proof of a result of Pajitnov from [P], stating the following:

Theorem 0.9: ([P]) Let θ be a nowhere vanishing integer closed one-form on a compact manifold M. Then $H^i_{\alpha\theta}(M) = 0$, for any α such that e^α is transcendental.

Chapter 4. LCK and LCS solvmanifolds. Main results

In Chapter 4 we investigate the possibility of computing the twisted cohomology of a solvmanifold only by looking at Lie algebra level. A solvmanifold is a quotient of a solvable Lie group G to a cocompact discrete subgroup Γ, $\Gamma\backslash G$. The motivation for looking at solvmanifolds was triggered by the fact that Inoue surfaces and their higher dimension analogues, OT's, are solvmanifolds (see [Be], [H], [Kas]). A solmanifold M may have particularities that give an isomorphism $H^i_{dR}(M) \simeq H^i_{dR}(\mathfrak{g})$. This is the case of completely solvable manifolds, as proved in [Hat]. In [Mi], this result is shown to hold for twisted cohomology as long as the one-form with respect to which the cohomology is considered is invariant under the action of G.

A new type of condition for having $H^i_{dR}(M) \simeq H^i_{dR}(\mathfrak{g})$, which is more general and includes the completely solvable case, has been done by Mostow in [Mos] and it is refered to as *the Mostow condition*. This means the following:

Definition 0.10: A solvmanifold $\Gamma\backslash G$ satisfies Mostow condition if $\mathrm{Ad}(\Gamma)$ and $\mathrm{Ad}(G)$ have the same Zariski closure in $\mathrm{GL}(\mathfrak{g})$ (here we denote by $\mathrm{GL}(\mathfrak{g})$ the group consisting of the linear isomorphisms of $\mathfrak{g}$, which are not necessarily Lie algebra automorphisms).

We start Chapter 4 by proving:

Proposition 0.11: ([AOT]) The Mostow condition is sufficient for a solvmanifold $\Gamma\backslash G$ in order to have the isomorphism $H^i_\theta(M) \simeq H^i_\theta(\mathfrak{g})$, for every G-invariant closed one-form θ.

We prove that Inoue surfaces $\mathcal{S}^0$ satisfy Mostow condition and the computation of twisted cohomology at Lie algebra level confirm the results obtained in Chapter 3 either by Mayer-Vietoris sequence or spectral sequences.

This legitimizes the question if OT manifolds satisfy Mostow condition, as a shortcut to compute the twisted cohomology of OT's. Of particular interest are *OT manifolds with one complex place*, which are proven in [OT] to carry LCK metrics. What we prove is that under a technical assumption OT manifolds with one complex place satisfy Mostow condition:

Theorem 0.12: ([AOT]) Let $X(K,U)$ be an Oeljeklaus-Toma manifold with precisely one complex place. Assume that there is no field T such that $\mathbb{Q} \subset T \subset K$ and T is totally real. Then $X(K, U)$ satisfies the Mostow condition.

We study an explicit example of OT manifold of type (2,1) which satisfies the assumption above and compute its twisted cohomology. Furthermore, we conclude that:

Corollary 0.13: For any natural number $s \geqslant 1$, there exists an Oeljeklaus-Toma manifold of type $(s, 1)$ satisfying the Mostow condition.

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