PREFACE

The purpose of this book is the study of non-self-adjoint boundary eigenvalue problems for first order systems of ordinary differential equations and n-th order scalar differential equations. The coefficients of the differential equations as well as the boundary conditions are allowed to depend polynomially, holomorphically or asymptotically on the eigenvalue parameter. The boundary conditions may contain infinitely many interior points and an integral term. With the boundary eigenvalue problem a bounded operator function is associated which consists of two components, the differential operator function and the boundary operator function. These operator functions depend in general nonlinearly on the eigenvalue parameter.

Various eigenfunction expansions are proved by the contour integral method under regularity conditions which originally were introduced by BIRKHOFF and STONE in case of λ-independent boundary condtitions. The calculation of the Fourier coefficients of these expansions is based on the theory of the inverses of holomorphic Fredholm operator valued functions which for the sake of completeness is included in this book. An important aspect of this theory is the representation of the principal parts of the inverses of these functions at their poles by root functions (eigenvectors and associated vectors) of the given operator functions and their adjoints. The proofs of the eigenfunction expansions are based on sharp asymptotic estimates of the resolvents (Green's functions) for large values of the eigenvalue parameter.

Our approach is based on functional analytic methods. The reader should be familiar with basic concepts of Banach spaces and Lebesgue integration and should have some knowledge about distributions. Whenever we use these basic results we give references so that the reader unfamiliar with these concepts can easily find them. Our main references to the basic topics are the monograph [KA] of T. KATO for Banach spaces, the monograph [HS] of E. HEWITT and K. STROMBERG for the theory of Lebesgue integration, and the monograph [HÖ2] of L. HÖRMANDER for the theory of distributions.

Each chapter ends with a short section containing historical notes.

Chapters I and II are concerned with preparations from functional analysis and Sobolev space theory. In Chapters III–V first order systems are considered, followed by n-th order equations in Chapters VI–IX. Since n-th order equations are reduced to first order systems, some of the results of Chapters III–V are needed in Chapters VI–IX. Chapter X contains applications to problems from physics and engineering.

The literature for n-th order linear differential equations and first order systems is vast, and the bibliography is only a selection of publications in this field. The list of notations and the index should help the reader to navigate through the text.