Preface

Introduction to Analysis is designed to bridge the gap between the intuitive calculus normally offered at the undergraduate level and the sophisticated analysis courses the student encounters at the senior or first-year-graduate level. Through a rigorous approach to the usual topics handled in one-dimensional calculus—limits, continuity, differentiation, integration, and infinite series—the book offers a deeper understanding of the ideas encountered in the calculus. Although the text assumes that the reader has completed several semesters of calculus, this assumption is necessary only for some of the motivation (of theorems) and examples.

The book has been written with two important goals in mind for its readers: the development of a rigorous foundation for the basic topics of analysis, and the less tangible acquisition of an accurate intuitive feeling for analysis. In the interest of these goals, considerable time is devoted to motivating and developing new concepts. Economy of space is often sacrificed so that ideas can be introduced in a natural fashion.

This 4th edition contains a number of changes recommended by the reviewers and users of earlier editions of the book. Chapter 0 contains introductory material on sets, functions, relations, mathematical induction, recursion, equivalent and countable sets, and the set of real numbers. As in the 3rd edition, the set of real numbers is postulated as an ordered field with the least upper bound property. Chapters 1 through 4 contain the material on sequences, limits of functions, continuity, and differentiation. Chapter 5 is devoted to the Riemann integral, rather than the Riemann–Stieltjes integral treated in the first edition. Chapter 6 treats infinite series, and Chapter 7 contains material on sequences and series of functions.

The exercise sets have been expanded and offer a selection of exercises with level of difficulty ranging from very routine to quite challenging. The starred exercises are of particular importance, because they contain facts vital to the development of later sections. The star is *not* used to indicate the more difficult exercises.

At the end of each chapter, you will find several PROJECTS. The purpose of a PROJECT is to give the student the necessary guidance to solve a substantial mathematical problem. A PROJECT is distinguished from an exercise in that the PROJECT is a multi-step problem which can be very difficult for the beginner without significant assistance. PROJECTS are sometimes used to cover material not included in the chapter discussions. For example, PROJECT 2.2 includes left- and right-hand limits. PROJECT 3.1 offers an approach to uniform continuity without the use of compactness. Other PROJECTS are used to generalize theorems and proofs in the text.

In the course of this exposition, a number of famous names are mentioned: Cauchy, Bolzano, Weierstrass, Riemann, Caratheodory, and others. A serious student should seek to know something about the persons who have made important contributions to analysis. The reader is urged to indulge in a little historical research when encountering the names of these people.

I would like to acknowledge the reviewers: Professor Lew Friedland, State University of New York; Professor Terry L. Herdman, Virginia Polytechnic Institute and State University; Professor Charles Himmelberg, University of Kansas; Professor Ken F. Kopfenstein, Colorado State University; and Professor James C. Magee, State University of New York.

My sincere appreciation is also extended to Robert J. Wisner, who, some 28 years ago, urged me to write the first edition of this book, and who has made many helpful suggestions relative to each of the three earlier editions; Jack N. Thornton, who as managing editor of Brooks/Cole Publishing Co. accepted and nurtured a fledgling author through the first and second editions; and to Jeremy Hayhurst, mathematics editor at Brooks/Cole Publishing Co., for his support and encouragement in the preparation of the third and fourth edition. Of course, this book would not be in your hands without the excellent work of Linda Loba who carried it and me through the stages of production. My thanks go to her also.

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