PREFACE

Modern Differential Geometry of Curves and Surfaces is a traditional text, but it uses the symbolic manipulation program *Mathematica*. This important computer program, available on PCs, Macs, NeXTs, Suns, Silicon Graphics Workstations and many other computers, can be used very effectively for plotting and computing. The book presents standard material about curves and surfaces, together with accurate interesting pictures, *Mathematica* instructions for making the pictures and *Mathematica* programs for computing functions such as curvature and torsion.

Although Curves and Surfaces makes use of *Mathematica*, the book should also be useful for those with no access to *Mathematica*. All calculations mentioned in the book can in theory be done by hand, but some of the longer calculations might be just as tedious as they were for differential geometers in the 19th century. Furthermore, the pictures (most of which were done with *Mathematica*) elucidate concepts, whether or not *Mathematica* is used by the reader.

The main prerequisite for the book is a course in calculus, both single variable and multi-variable. In addition, some knowledge of linear algebra and a few basic concepts of point set topology are needed. These can easily be obtained from standard sources. No computer knowledge is presumed. In fact, the book provides a good introduction to *Mathematica*; the book is compatible with both versions 2.2 and 3.0. For those who want to use Curves and Surfaces to learn *Mathematica*, it is advisable to have access to Wolfram's book Mathematica for reference. (In version 3.0 of *Mathematica*, Wolfram's book is available through the help menus.)

Curves and Surfaces is designed for a traditional course in differential geometry. At an American university such a course would probably be taught at the junior-senior level. When I taught a one-year course based on Curves and Surfaces at the University of Maryland, some of my students had computer experience,

others had not. All of them had acquired sufficient knowledge of *Mathematica* after one week. I chose not to have computers in my classroom because I needed the classroom time to explain concepts. I assigned all of the problems at the end of each chapter. The students used workstations, PCs and Macs to do those problems that required *Mathematica*. They either gave me a printed version of each assignment, or they sent the assignment to me by electronic mail.

Symbolic manipulation programs such as *Mathematica* are very useful tools for differential geometry. Computations that are very complicated to do by hand can frequently be performed with ease in *Mathematica*. However, they are no substitute for the theoretical aspects of differential geometry. So **Curves and Surfaces** presents theory and uses *Mathematica* programs in a complementary way.

Some of the aims of the book are the following.

- To show how to use *Mathematica* to plot many interesting curves and surfaces, more than in the standard texts. Using the techniques described in **Curves and Surfaces**, students can understand concepts geometrically by plotting curves and surfaces on a monitor and then printing them. The effect of changes in parameters can be strikingly portrayed.

- The presentation of pictures of curves and surfaces that are informative, interesting and accurate. The book contains over 400 illustrations.

- The inclusion of as many topics of the classical differential geometry and surfaces as possible. In particular, the book contains many examples to illustrate important theorems.

- Alleviation of the drudgery of computing things such as the curvature and torsion of a curve in space. When the curvature and torsion become too complicated to compute, they can be graphed instead. There are more than 175 miniprograms for computing various geometric objects and plotting them.

- The introduction of techniques from numerical analysis into differential geometry. *Mathematica* programs for numerical computation and drawing of geodesics on an arbitrary surface are given. Curves can be found numerically when their torsion and curvature are specified. See the pictures in Section 6.4.

- To place the material in perspective through informative historical notes. There are capsule biographies with portraits of over 75 mathematicians and scientists.

- To introduce interesting topics that, in spite of their simplicity, deserve to be better known. I mention triply orthogonal systems of surfaces (Chapter 29), Björling's formula for constructing a minimal surface containing a given plane curve as a geodesic (Chapter 33) and canal surfaces and cylides of Dupin as Maxwell discussed them (Chapter 35).

- To develop a dialect of *Mathematica* for handling functions that facilitates the construction of new curves and surfaces from old. For example, there is a simple program to generates a surface of revolution from a plane curve.

- To provide explicit definitions of curves and surfaces. Over 300 *Mathematica* definitions of curves and surfaces can be used for further study.

The approach of **Curves and Surfaces** is admittedly more computational than is usual for a book on the subject. For example, Brioschi's formula for the Gaussian curvature in terms of the first fundamental form can be too complicated for use in hand calculations, but *Mathematica* handles it easily, either through computations or through graphing the curvature. Another part of *Mathematica* that can be used effectively in differential geometry is its special function library. For example, non-standard spaces of constant curvature can be defined in terms of elliptic functions and then plotted.

Frequently, I have been asked if new mathematical results can be obtained by means of computers. Although the answer is generally no, it is certainly the case that computers can be an effective supplement to pure thought, because they allow experimentation and the graphs provide insights into complex relationships. I hope that many research mathematicians will find **Curves and Surfaces** useful for that purpose. Two results that I found with the aid of *Mathematica* are the interpretation of torsion in terms of tube twisting in Chapter 9 and the construction of a conjugate minimal surface without integration in Chapter 31. I have not seen these results in the literature, but they may not be new.

The programs in the book, as well as some descriptive *Mathematica* notebooks, will eventually be available on the web. In the meantime the programs and notebooks can be down-loaded via anonymous ftp from **bianchi.umd.edu**, which is the computer in my office. I have also made images of many curves and surfaces that are displayable using the programs acrospin and geomview, as described in Appendix D. These files are also available from **bianchi.umd.edu**. On the ftp site *Maple* definitions of curves and surfaces are given as well. My email address is **gray@bianchi.umd.edu**.