Foreword

It was in the late forties that the theory of Boolean algebras was given the first practical application to the study of switching circuits. This approach to a problem of techniques became possible due to the fact that, as switching circuits are generally made up of two-state devices, they may be studied by means of a mathematical apparatus using bivalent variables. This apparatus was found to be the two-element Boolean algebra.

It is a natural idea to use bivalent variables whenever we are faced with problems involving situations with only two possible outcomes. Such problems of "binary decisions" are frequently found in operations research, theory of graphs, combinatorial mathematics, etc. The importance and width of this class of problems was first pointed out by G. B. DANTZIG in 1957.

Meanwhile many papers have been published on those topics and the research is in full progress. Some of the works in the field apply *Boolean techniques*, that is techniques actually using properties of Boolean algebra, while other research is mainly combinatorial.

The present book deals almost exclusively* with applications of Boolean techniques (in the above sense) to a field which, in lack of a better term, was called "operations research and related areas".

The main tools which will be described and employed in the book are: Boolean matrix calculus, Boolean equations and pseudo-Boolean programming.

The core of the book is pseudo-Boolean programming, a method for solving bivalent (0, 1) programs which was developed by I. ROSENBERG and the authors in 1963, using an idea of R. FORTET. The method, which, in fact, is a combination of R. BELLMAN's principle of dynamic programming with Boolean procedures, is presented here in an improved form, due to the present authors, along with various applications of it. Such applications were made, in addition to the authors, by I. ROSENBERG, Y. INAGAKI and K. SUGINO, U. S. R. MURTY, G.-B. IHDE, J. KRÁL et al.; these contributions are based on the first (unimproved) version of pseudo-Boolean programming.

As there exists a wide literature on Boolean methods in switching theory, this topic was not included in our book, except for an application of pseudo-Boolean programming to certain minimization problems of switching theory. We have also omitted non-Boolean approaches

* With the exception of §§ 1—4 in Ch. III and of §§ 1—2 in Ch. V.

to problems of mathematical programming with bivalent variables. However, we have included a bibliography of books on switching theory.

Our book is intended for people interested in operations research, on the one hand, and for those interested in applications of Boolean algebras, on the other hand. Therefore the content will be as follows.

In the first chapters of the book we present the necessary mathematical background: Boolean functions and equations, pseudo-Boolean functions (i.e. real-valued functions with bivalent (0, 1) variables), pseudo-Boolean equations and inequalities, pseudo-Boolean programming.

The method of pseudo-Boolean programming enables the solution of linear and nonlinear bivalent programs as well as of various generalizations including integer polynomial programming.

The last chapters are devoted to applications to the theory of graphs, networks, partially ordered sets, sequencing problems, assignment and transportation problems, time-table scheduling, optimal plant location, minimization problems in switching circuits, etc.

The book includes also an Appendix written by I. ROSENBERG on a generalization of pseudo-Boolean programming.

Besides the main bibliography, two supplementary bibliographies (A and B) on Boolean equations and switching theory are also included. Since the number of papers on switching algebra is tremendous, the latter bibliography contains only books pertinent to the field.

Any omission in these bibliographies is due to the ignorance of the authors and is not to be considered as the result of a valuation.

We have tried to make this book as self-contained as possible. The reader is supposed to be familiar with the usual symbols of set theory; we mention here that notations like $A \cup B$ or $\bigcup_{i=1}^{n} A_i$ mean set-theoretical joins, while $a \cup b$ or $\bigcup_{i=1}^{n} a_i$ denote Boolean disjunction of elements.

Formulas, theorems, lemmas, etc., are numbered separately in each chapter. References to formulas, theorems, lemmas, etc. in the same chapters indicate only their numbers, while references to other chapters indicate the number of the corresponding chapter (for instance, formula (II.13), Theorem IV.1, etc.).

The authors would much appreciate receiving observations from the readers.

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