Preface

The interpolation is one of the easiest and most important procedure for approximating a real-valued function under both aspects, theoretical and practical. As many of physical processes are modelled by functions, the problem which appears is to approximate a function based on some limited information (data) about the corresponding process. Usually, such information are the values of the modelled function or of certain of its derivatives at some given points, obtained by different measurements.

The aim of the present book is to study the interpolation problems in one dimensional and multidimensional cases. Taking into account the definition of an interpolation operator used here (for $\mathcal{B}$ a linear space of real-valued functions defined on a domain $\Omega \subset \mathbb{R}^n$, $\mathcal{A} \subset \mathcal{B}$ and $\Lambda := \{\lambda_i \mid \lambda_i : \mathcal{B} \to \mathbb{R}, i = 1, ..., N\}$ a set of linear functionals, $P : \mathcal{B} \to \mathcal{A}$ for which $\lambda_i(Pf) = \lambda_i(f)$, $i = 1, ..., N$, $f \in \mathcal{B}$, is called an interpolation operator with regard to Λ), one remarks that an interpolation operator essentially depends on the sets $\mathcal{A}$ and Λ. Therefore, the book is organized with respect to this two sets. For $\mathcal{A}$ are taken the sets: $\mathbb{P}$ (polynomial functions), $\mathcal{S}$ (natural spline functions) and $\mathcal{R}$ (rational functions), while Λ was considered as Lagrange, Hermite, respectively, Birkhoff type sets of functionals.

The structure of this book is the following.

Chapter 1 is devoted to some preliminary notions.

Chapter 2 treats the univariate case of interpolation. The polynomial, spline and Shepard interpolation operators are discussed. Also, the remainders of the corresponding interpolation formulas are studied. As an extension of the interpolation procedure, the least square approximation is briefly presented.

Chapter 3 presents the multivariate interpolation operators. There are introduced two Sard spaces and extensions of Peano's theorem for two and three dimensional cases, which are used for studying the remainder terms in multivariate interpolation formulas. Most of the results have as starting point the Gordon's algebraic approach [113]. The lattices of projectors are also discussed. Next, the multivariate interpolation operators are treated for regularly domains (rectangular, simplex) as well as for arbitrary domains, with a special emphasis on Shepard operators.

Chapter 4 contains some applications of interpolation operators and of interpolation formulas in surfaces generation, numerical integration of functions, numerical methods for nonlinear equations in $\mathbb{R}$, generated by inverse interpolation and some other special applications.

The content of this book is essentially based on the work and results of the authors, Gheorghe Coman and his seven doctorands (Daniela Cătinaş, Marius Birou, Alexandra Oprişan, Cristina Oşan, Ioana Pop, Ildiko Somogyi and Ioan Todea).

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