Preface

In this book we give an elementary point of view of some classical topics in the theory of holomorphic functions of several complex variables. There are some phenomena which arise in higher dimensions and are of particular interest. Among them we mention the following:

• The Riemann mapping theorem concerning the conformal equivalence of simply connected domains in the complex plane does not hold in higher dimensions.

• A domain Ω in $\mathbb{C}^n$, $n \geq 2$, is not necessarily a domain of holomorphy, which means that Ω need not be the maximal domain of definition for some holomorphic function.

• The holomorphic functions of several complex variables cannot have isolated singularities (any isolated singularity is removable).

The first chapter begins with an introduction to the theory of holomorphic functions in higher dimensions. We present equivalent ways of defining holomorphic functions and we give basic properties of these functions such as the Cauchy integral formula, the maximum principle, etc. We shall also present the topological and metric structures of $H(\Omega, \mathbb{C})$, where Ω is an open subset of $\mathbb{C}^n$, and we give a detailed proof of the well known result due to Hartogs concerning the equivalence between separate holomorphy and (global) holomorphy. We also discuss an analogous result of Forelli concerning functions which are holomorphic on complex lines through the origin.

The second chapter deals with basic properties of holomorphic mappings. A key role in our discussion is played by the Cartan uniqueness results, which allow us to determine the biholomorphic automorphisms of the unit ball and the unit polydisc. We also present a theorem of Cartan concerning the convergence of sequences of automorphisms of a bounded domain.

In the third chapter we study two very important notions in the theory of holomorphic functions, namely the notions of domain of holomorphy and pseudoconvexity. In fact, they are equivalent. We shall also study two types of pseudoconvexity, i.e. Hartogs pseudoconvexity and Levi pseudoconvexity.

The last chapter is devoted to the study of some subclasses of biholomorphic mappings on the unit ball and the unit polydisc in $\mathbb{C}^n$, which involve geometrical as well as analytical properties. We shall present classical and also recent results concerning starlike and convex mappings, and we shall investigate the connection between the notion of kernel convergence of domains in $\mathbb{C}^n$ and biholomorphic mappings.

The book is intended as an introductory course in several complex variables for graduate students; the last chapter may also be of interest to research mathematicians. The prerequisites are a course in complex analysis of one variable, a course in topology and measure theory, and some basic notions of functional analysis. We have included some exercises throughout.

The main sources used in the preparation of chapters I-III are some of the well known books in several complex variables, including [Cha, vol. II], [Gra-Fri], [Kra], [Rud2], and especially [Gun], [Hör], [Ran], [Nar], [Kau-Kau]. The last chapter, except for the sections 4.5 and 4.6, is based on the recent book [Gr-Ko2].

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