Introduction

The developments in the recent years of the potential theory emphasized a class of functions larger than that of excessive functions (i.e. the positive superharmonic functions from the classical potential theory associated with the Laplace operator), namely the *strongly supermedian functions*. It turns out that a positive Borel function will be strongly supermedian if and only if it is the infimum of all its excessive majorants. Apparently, these functions have been introduced by J.F. Mertens and then they have been studied mainly by P.A. Meyer, G. Mokobodzki, D. Feyel and recently by P.J. Fitzsimmons and R.K. Getoor.

The aim of this book is among others to develop a potential theory appropriate to this new class of functions. Although our methods are analytical, we present also the probabilistic counterparts from the Markov processes theory.

The natural frame in which this theory is settled is given by a sub-Markovian resolvent of kernels on a Radon measurable space. After a possible extension of the space, such a resolvent becomes that one associated with a right process on a Radon topological space, not necessary locally compact and without existing a reference measure.

Intimately related to the excessive functions we present certain basic tools of the theory: the Ray topology and compactification, the fine carrier and the reduction operation on measurable sets. We examine different types of negligible sets with respect to a finite measure λ: the λ-polar, λ-semipolar and λ-mince sets. We take advantage of the cone of potentials structure for both excessive functions and measures.

A notable part of the theory is the study of the *subordination* for sub-Markovian resolvents, that is to investigate the sub-Markovian resolvents which are dominated by a given one. This operation extends the localization on finely open sets (which in the classical case means to pass from the superharmonicity on the whole space to the superharmonicity on a finely open subset), and also the subordination generated by a measure μ (which means to pass from the Laplace operator Δ to the operator $\Delta - \mu$). In terms of Markov processes, the subordinate resolvents are those associated with the subprocesses (induced by multiplicative functionals) of a given right process.

Another important part of the theory is the so called *Revuz correspondence* which substitutes the correspondence between measures and their associated Green potentials from the classical potential theory. This procedure deepens the probabilistic results on the homogenous random measures and the positive additive functionals.

The last chapter is devoted to the study of the *weak duality* between two sub-Markovian resolvents on a common Lusin topological space, with respect to a given measure. The context covers the probabilistic frame of two (Borel) right processes in weak duality. A special case is given by a sub-Markovian resolvent of kernels satisfying the strong sector condition (or equivalently by the resolvent of kernels of a quasi-regular semi-Dirichlet form).

Each chapter, except for the second one, is completed with a section reserved for the *probabilistic interpretations* of the obtained analytical results.

The book intends to be self-contained (as much as possible). Chapter 2 has a special position, presenting some necessary notions and results concerning the cones of potentials and H-cones. However, the proofs, being rather technical, are presented in Appendix (and may be skipped at a first reading). Basic facts on the probabilistic potential theory associated with a right process are also collected in the Appendix and we refer to the monograph [Sh 88] of M. Sharpe for more details. The Appendix is completed with some other complements from measure and capacity theory and the semi-Dirichlet forms.

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