# Preface

## Basic philosophy

Algebra, as we know it today, consists of a great many ideas, concepts and results. A reasonable estimate of the number of these different "items" would be somewhere between 50 000 and 200 000. Many of these have been named and many more could (and perhaps should) have a "name", or other convenient designation. Even the nonspecialist is quite likely to encounter most of these, either somewhere in the literature distinguised as a definition or a theorem or to hear about them and feel the need for more information. If this happens, one should be able to find at least something in this Handbook; and hopefully enough to judge whether it is worthwhile to pursue the quest at least. In addition to the primary information references to relevant articles, books or lecture notes should help the reader to complete his understanding.

To make this possible we have provided an index which is more extensive than usual, and not limited to definitions, theorems and the like.

For the purposes of this Handbook, algebra has been defined more or less arbitrarily as the union of the following areas of the Mathematics Subject Classification Scheme:

- 20 (Group theory)
- 19 ($K$-theory; this will be treated at an intermediate level)
- 18 (Category theory and homological algebra; including some of the uses of category in computer science, often classified somewhere in section 68)
- 17 (Nonassociative rings and algebras; especially Lie algebras)
- 16 (Associative rings and algebras)
- 15 (Linear and multilinear algebra, Matrix theory)
- 13 (Commutative rings and algebras; here there is a fine line to tread between commutative algebras and algebraic geometry; algebraic geometry is definitely not a topic that will be dealt with in this Handbook; there will, hopefully, one day be a separate Handbook on that topic)
- 12 (Field theory and polynomials)
- 11 The part of that also used to be classified under 12 (Algebraic number theory)
- 08 (General algebraic systems)
- 06 (Certain parts; but not topics specific to Boolean algebras as there is a separate three-volume Handbook of Boolean Algebras)

## Planning

Originally (1992), we expected to cover the whole field in a systematic way. Volume 1 would be devoted to what is now called Section 1 (see below), Volume 2 to Section 2, and so on. A detailed and comprehensive plan was made in terms of topics which needed to be covered and authors to be invited. That turned out to be an inefficient approach. Different authors have different priorities and to wait for the last contribution to a volume, as planned originally, would have resulted in long delays. Therefore, we have opted for a dynamically evolving plan. This also permits to take new developments into account.

This means that articles are published as they arrive and that the reader will find in this third volume articles from five different sections. The advantages of this scheme are two-fold: accepted articles will be published quickly and the outline of the series can be allowed to evolve as the various volumes are published. Suggestions from readers both as to topics to be covered and authors to be invited are most welcome and will taken into serious consideration.

The list of the sections now looks as follows:

Section 1: Linear algebra. Fields. Algebraic number theory
Section 2: Category theory. Homological and homotopical algebra. Methods from logic (algebraic model theory)
Section 3: Commutative and associative rings and algebras
Section 4: Other algebraic structures. Nonassociative rings and algebras. Commutative and associative rings and algebras with extra structure
Section 5: Groups and semigroups
Section 6: Representations and invariant theory
Section 7: Machine computation. Algorithms. Tables
Section 8: Applied algebra
Section 9: History of algebra

For a more detailed plan (2002 version), the reader is referred to the Outline of the Series following this preface.

## The individual chapters

It is not the intention that the handbook as a whole can also be a substitute undergraduate or even graduate, textbook. The treatment of the various topics will be much too dense and professional for that. Basically, the level is graduate and up, and such material as can be found in P.M. Cohn's three volume textbook "Algebra" (Wiley) will, as a rule, be assumed. An important function of the articles in this Handbook is to provide professional mathematicians working in a different area with sufficient information on the topic in question if and when it is needed.

Each chapter combines some of the features of both a graduate-level textbook and a research-level survey. Not all of the ingredients mentioned below will be appropriate in each case, but authors have been asked to include the following:

- Introduction (including motivation and historical remarks)
- Outline of the chapter
- Basic concepts, definitions, and results (proofs or ideas/sketches of the proofs are given when space permits)
- Comments on the relevance of the results, relations to other results, and applications
- Review of the relevant literature; possibly supplemented with the opinion of the author on recent developments and future directions
- Extensive bibliography (several hundred items will not be exceptional)

**The future**

Of course, ideally, a comprehensive series of books like this should be interactive and have a hypertext structure to make finding material and navigation through it immediate and intuitive. It should also incorporate the various algorithms in implemented form as well as permit a certain amount of dialogue with the reader. Plans for such an interactive, hypertext, CD-Rom-based version certainly exist but the realization is still a nontrivial number of years in the future.

Kvoseliai, July 2003 Michiel Hazewinkel

Kaum nennt man die Dinge beim richtigen Namen,
so verlieren sie ihren gefährlichen Zauber

(You have but to know an object by its proper name
for it to lose its dangerous magic)

E. Canetti