To the memory of my father

Introduction

Many important "*real world*" problems are modelled by systems of linear equalities or inequalities, in classical or least-squares formulations. These systems are usually big and ill-conditioned, thus iterative solvers, in classical or preconditioned versions are well suited. The principal aim of the present research monograph booklet is to describe some classes of such iterative algorithms, successfully used today.

We start the presentation in Chapter 1 with some very important practical problems which give rise to least-squares formulations of systems of linear equalities or inequalities. In Chapters 2 and 3 we describe the well-known Kaczmarz's projections algorithm, together with some row and block-row extensions. In Chapter 4 we present preconditioning techniques for general linear least-squares problems. We start with the description of a general preconditioning technique, "inspired" from the case of square nonsingular system in connection with the so called "oblique projections" method. In the second part of Chapter 3, we describe an approximate rows (and columns) orthogonalization method due to Z. Kovarik and the way in which it can be applied as preconditioner. Chapter 5 presents the "*reflections*" algorithm, due to G. Cimmino and some of its extensions to consistent least-squares problems. Chapter 6 is devoted to systems of linear inequalities, reformulated as quadratic optimization problems. First of all we consider consistent systems of linear inequalities for which we describe the original " *relaxation* " method due to C. Hildreth and a generalization based on Kaczmarz-like projections on the system's hyperplanes, due to Y. Censor and A. Lent. In the last part of the chapter we consider a least - squares formulation of an inconsistent system of linear inequalities and present the algorithm of S. Han for approximating such kind of solutions.

In Chapter 7 we describe the classical version of conjugated gradient algorithm, for full-rank least-squares problems and some of its generalizations for arbitrary ones obtained by Kammerer-Nashed and Tanabe. In Chapter 8 we present the Schulz-Hotelling-Bodewig algorithm for approximating the inverse of a square nonsingular matrix, but from the view-point of preconditioning for Kaczmarz's or (CG) methods. From the same view point we describe in the second part of Chapter 8 a generalization of the above algorithm due to A. Ben-Israel, which approximates the Moore-Penrose pseudoinverse of a matrix. The monograph ends with Chapter 9 in which we present numerical experiments with two of the algorithms described in the previous chapters.

Beside the above mentioned classical and well-known facts the book contains many original results and algorithms obtained by the author. For each method we present the necessary theoretical considerations together with remarks and comments concerning their practical implementation. We preferred to underline the basic ideas and not to go into too cumbersome and difficult theoretical considerations (for which complete references have been done each time). We have also considered during the presentation only the "*real numbers context*", although almost all the algorithms and results are true also in the complex case.

We hope that the booklet will be useful for scientists interested in numerical approximations of solutions for " *real world* " problems.

At the end, I would like to thank to all those from which I obtained moral and financial support during the preparation of the manuscript, especially to my friend Mr. Dan Maiorescu, Technical Manager of "Develop4@ll" Paris, France.

Constanta, May 2002

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