Current Topics in Continuum Mechanics. I,
Edited by Lazăr Dragoş,

Foreword

It is well known the huge importance that mechanics had upon the development of mathematics and at the same time the role played by mathematics in the establishment of the mechanics laws. It may be said indeed that these two sciences conditioned each other, the outstanding mathematicians being also great mechanicians and conversely. A first example is Isaac Newton (1643–1727) who formulated the principles of mechanics, founded the differential and integral calculus just necessary to him for characterizing the functions with continuous variation that define the movement.The development of his theory allowed Euler in 1736 to write for the first time the equation that was called after Newton's name. Newton's work "Philosophiae naturalis principia mathematica" was published in 1687. Among Newton's results in mathematics the binomial formula and Newton-Cotes formula has to be be remind. In mechanics, the deduction of the law of universal attraction, the potential theory, the first elements of celestial mechanics are due to him.

Another peak in science was Leonhard Euler (1707–1783) – a great mathematician as well as a great mechanician. In mathematics Euler had an essential contribution to the development of the differential and integral calculus, wrote the first differential equations and together with J. Lagrange put the basis os the variational calculus. In mechanics he let fundamental results, specifically: Newton's equation (in the work "Mechanica sive motus scientia analitice exposita"), introduced the concept of pressure and deduced the equations of ideal fluids, established the theory of the movement of the solid body with a fixed point and put the basis of hydraulics theory.

Simultaneously, another mathematician–mechanician, Jean le Rond d'Alembert (1717–1783) formulated the fundamental theorem of algebra, solved the equation of wave propagation, established a criterion for the convergence of numerical series and in mechanics formulated the principle that got his name, wrote the first treatise of mechanics ("Traité de Dynamique", Paris, 1743). In the domain of fluid mechanics he also deduced the continuity equation and formulated the paradox that received his name.

We continue with Joseph-Louis Lagrange (1736–1813) whose results in mathematical analysis are: the finite increments theorem, the extension of Taylor's formula to the functions of many variables, extremum problems for functions of many variables, interpolation relationships. He also established the method of variation of parameters and the Lagrange-Charpit method for the integration of first order partial differential equations. Inspired by mechanics, like Euler he is the creator of the variational calculus. In mechanics, Lagrange's most important work is "Mécanique analytique" (1788), in which he reduces the mechanics problems to mathematical analysis problems. Lagrange applied the perturbation theory to the problem of the three bodies, put the basis of the dynamics of the heavy rigid body. In hydrodynamics introduced the variables that received after then his name and deduced the motion equations with respect to these variables.

Another example of mathematician who extracted problems for study from mechanics was Simeon-Denis Poisson (1781–1840). Well known for his researches upon the harmonic functions and upon the singular solutions of differential equations, he also studied the solid body movement and the long axes invariability of the planetary orbits. We have to remind his researches concerning the theory of heat propagation and those related to the probability theory.

Augustin-Louis Cauchy who lived and created in the 19th century is claimed equally by mathematicians and mechanicians. He remains without any doubt the most remarkable example of mathematician-mechanician, or more correctly mechanician-mathematician. We have to mention first his results in mathematical analysis: the sequence theory (fundamental sequences, Cauchy sequences), the properties of derivable functions (Cauchy theorem), the characteristics method for the integration of first order partial differential equations and the existence and uniqueness theorem for the second order partial differential equations (Cauchy-Kowalevskaia theorem). It is the founder of the complex analysis (the differential theory of the holomorphy, the residue theory). But as valuable would be these results in mathematics, we must emphasize that the deduction of the motion (and equilibrium) equations in the continuum mechanics is his brilliant result because all there is known, even nowadays, about the deformation and motion of fluids, solids and plasma, is based on these equations.

Up to the end of the 19th century we find Henri Poincaré (1854–1912). He had a vast work (over 500 papers) in mathematics, mechanics and in engineering too. He is the creator of the dynamical systems and of singular integral equations theories. Due to his works mechanics and especially celestial mechanics registered an enormous progress.

At the beginning of the 20th century, Tullio Levi-Civita (1873–1941) contributed with fundamental results to the development of the differential geometry, fluid mechanics and to the mechanics of systems of points.

Let us mention Vito Volterra (1860–1940) known due to his results concerning the theories of integral equations and partial differential equations. He introduced the concept of functional, being one of the founders of functional analysis. In mechanics he studied the theory of elasticity and the theory of dislocations.

Finally let us remind about the great geometer - Elie Cartan (1869–1951) with fundamental contributions to the Lie group theory and who has basic results in the theory of canonical systems and integral invariants and the great algebrist Garrett Birkhoff who let an excellent critical study about basic results in hydrodynamics.

I do not want to end this presentation without reminding that the Russian mathematician I.G.Ghelfand understood very well the importance of mechanics as source of mathematical theories, writing down in "Uspei matematicescoi nauki" that the forces that would incite or determine the development of functional analysis are the continuum mechanics and quantic mechanics. Without bringing new elements from outside it, mathematics would become sterile. The same idea we find in Vranceanu's words : "mechanics is the window through what mathematics is regarding to the world". The great mathematicians understood very well this role. That is why in all respectable mathematics faculties all over the world, mechanics is found among the fundamental disciplines of study.

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Introduction

The high level didactic activity cannot be separated from a sustained research activity. The most efficient framework in which these activities may simultaneously develop has been acknowledged to be the scientific seminars. The understanding of the importance of this form of interweaving between teaching and research determined outstanding professors to organize two scientific seminars within the Department of Mechanics and Equations belonging to the Faculty of Mathematics of the University of Bucharest. The first one is the Seminar of Fluid Mechanics and Applied Mathematics founded by the regretted professors Victor Vâlcovici and Caius Iacob in 1950 and the second is the Seminar of Deformable Media organized by professors Gr. C. Moisil and Nicolae Cristescu.

The topics of these seminars cover various subjects from general mechanics and mechanics of continua up to the most refined mathematical methods used in mechanics.

Within these meetings original works of high level are presented, the results of Ph.D. thesis are debated and new directions in research are studied. The results communicated are going to be published in national and international journals of prestige. Unfortunately, the publication of the papers in journals or in conference proceedings often is restricted to a limited number of pages imposing sometimes the abridgement of the text. That is why the issue of a volume series, entitled Current Topics in Continuum Mechanics, that would include original papers presented at the seminars and translated in a language of international circulation, has been necessarily imposed.

The present volume that opens a series we hope to be able to sustain financially comprises 6 chapters arranged following the authors' alphabetical order. Further, we briefly present their contents.

i) *Numerical and qualitative study of fluid jets* – Adrian Carabineanu

The model Helmholtz-Kirchhoff associated to the flow of a fluid jet into the atmosphere is studied, the problem being reduced to a system of nonlinear equations. Previous author's results concerning the existence of the solution are reanalyzed. Then, a semi-inverse method for the calculation of the free lines of the jet whose walls consist of either arcs of circle or semi-infinite straight lines is presented and discussed.

ii) *Large elasto-plastic deformations for crystalline materials* – Sanda Cleja-Ţigoiu

A constitutive framework for the description of the behaviour of elasto-plastic materials is presented and the role of the relaxed and isocline current local configurations is put in evidence. Then, some classes of anisotropic materials (transversal isotropic, orthotropic) that model composite materials are presented. The role of the plastic spin upon the description of the behaviour of such materials is studied. There are formulated variational inequalities for describing rate type quasi-static problems. Finally it is evidenced the behaviour of these materials subjected to movements with homogeneous deformations and is developed a study regarding the occurrence of the possible bifurcations of the solutions.

iii) *Analytical and numerical approach to Richards' equation* – Stelian Ion, Gabriela Marinoschi and Dorin Marinescu

In this chapter there are presented analytical and numerical solutions of boundary-value problems with initial conditions attached to Richards' nonlinear parabolic equation. There are investigated some mathematical models corresponding to unsaturated soils characterized by certain hydraulic properties (Gardner's exponential model, Broadbridge's model) for which analytical solutions may be obtained. The mathematical properties of the numerical solutions for other models (e.g. Philip's model and van Genuchten's model) are discussed. Numerical results are provided and the graphics are represented using the software "Gnuplot".

iv) *Creep, damage and failure around rectangular-like galleries* – Iuliana Paraschiv-Munteanu

The chapter presents the determination of the strains and displacements around a cylindrical horizontal cavity (with noncircular section). For this purpose the rock is modeled by the means of a visco-plastic model proposed by N. Cristescu. There are discussed problems related to the compressibility and dilatancy around the cavity, the damage and/or the wall closure in time.

v) *Functional framework for linear variational equations* – Ioan Roşca

The paper presents a unitary approach of abstract variational equations with applications to the study of the boundary value problems for partial derivative equations and systems of equations. A new proof of the existence and uniqueness of the variational equations with restrictions is given and the role of F. Brezzi's inf-sup condition is emphasized. Also, an elementary proof of J.L. Lions' regularity lemma is presented and a simple proof of Korn's inequality is obtained. The results are particularized to the study of boundary-value problems in mechanics.

vi) *Uniqueness and asymptotic stability for some fluids of differential type* – Victor Ţigoiu

The paper gives a definitive response to the problem raised by D.D. Joseph regarding "the existence of polynomial fluids" (at least for the fluids of third degree). Indeed, without using the well known theorem of functionals representation (of Coleman and Noll) and on the basis of *à priori* estimations obtained in the proof of the uniqueness of the problem with initial and boundary conditions, it is proved that the rest state is asymptotic stable, regardless the sign of the second Rivlin-Ericksen tensor (for the fluids of third degree). In consequence, at least this class of polynomial fluids may exist in the sense of Joseph's assertion.

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