Preface

The researchers in Aerodynamics know that there is not a unitary method of investigation in this field. The first mathematical model of the airplane wing, the model meaning the integral equation governing the phenomenon, was proposed by L. Prandtl in 1918. The integral equation deduced by Prandtl, on the basis of some assumptions which will be specified in the sequel, furnishes the circulation $C(y)$ (see Chapter 6). Using the circulation, one calculates the lift and moment coefficients, which are very important in Aerodynamics. The first hypothesis made by Prandtl consists in replacing the wing by a distribution of vortices on the plan-form D of the wing (i.e. the projection of the wing on the plane determined by the direction of the uniform stream at infinity and the direction of the span of the wing). Since such a distribution leads to a potential flow in the exterior of D and the experiences show that downstream the flow has not this character, Prandtl introduces as a supplementary hypothesis another vortices distribution on the trace of the domain D in the uniform stream. The first kind of vortices are called tied vortices and the second kind of vortices are called free vortices. On the basis of this model one developed later the main theories of Aerodynamics namely the lifting surface theory (after 1936, more precisely in 1950, when Multhopp gave the equation of this theory), the lifting surface theory for the supersonic flow (after 1946) and the lifting theory for oscillatory wings and surfaces for the subsonic, sonic and supersonic flow (after 1950). In the framework of the last theory the wing is replaced by doublets distributions. From a physical point of view, there is no reason for replacing the wing with vortices or doublets distributions. It is true that the vortices are detaching from the wing, but these are effects , not causes of the presence of the wing. The fact that these replacements lead to correct results shows how subtle was Prandtl's intuition. We specify that the distributions on D and its trace do not result from the equations of motion (they have been introduced outside the mathematical model). Taking into account this inconvenient, we have shown in [5.7] how it can be removed. We have to consider that the wing and the fluid constitute an interacting material system. If we want to study the fluid flow, then according to Cauchy's stress principle (the principle of the internal forces; see for example [1.11], p.35), we have to assume that there exists a forces distribution on the boundary, which has against the fluid the same action like the wing itself. We shall replace therefore the wing with a forces distribution instead of a vortices, sources or doublets distribution and we shall find out the density of this distribution such that it should have the same action against the fluid like the wing itself. We shall proceed by imposing to the fluid flow determined by the forces distribution to satisfy the slipping condition on the wing, condition which is also satisfied by the flow determined by the wing. In this way it follows an integral equation for determining the forces density. This equation constitutes the mathematical model for the wing we have in view. This method is an unitary one and it is based only on the classical principles of mechanics (in fact, Cauchy's stress principle). It may be applied to all configurations: see [5.7] for the wing in a subsonic stream, [8.4] for the wing in a supersonic stream, [10.15], [10.16], [10.17] for the oscillatory wings in subsonic, sonic or supersonic stream etc. All these results are given in this book (see chapters 5, 8, 10, 11). We called this method (in[5.7]): the fundamental solutions method. It may be utilized to all cases in which one can calculate the fundamental solutions of the equations of motion. We have to notice that in the framework of this method, the existence of the vortices downstream the wing follows from the model (i.e. from the equations of motion) and it must not be introduced artificially. In the sequel we shall present some of the models of aerodynamics. For two-dimensional configurations, in a subsonic stream, the models are one-dimensional singular integral equations considered in the sense of Cauchy's principal value. One may integrate analytically only the equation of thin profiles in a free stream. For other geometries one determines numerical solutions with the aid of Gauss-type quadrature formulas (see Chapter 3). For three-dimensional wings in a subsonic stream, the models are two-dimensional integral equations with strong singularities, which are defined in the sense of Finite Part (see Chapter 5). For other geometry (for example the wing in ground effects) the models are generalized equations. All these models are solved only numerically. For the wing in a free stream, Multhopp's method is available. In this book we introduce a more general method – the quadrature formulas method. In the last part of Chapter 5 one presents the theory of low aspect wings which was extended by the author to the general case of asymmetrical wings. The lifting line theory may be deduced from the lifting surface theory with the aid of Prandtl's assumptions (6). This theory is developed by presenting analytical and numerical methods for solving Prandtl's equation; one considers also extensions of this theory, all the methods representing one-dimensional integral-differential equations. The author shows how these equations may be reduced to integral equations with strong singularities and for this type of singularities he gives a Gauss-type quadrature formula, which allows the equation to be reduced to a linear algebraic system which is solved numerically. This method, which is very general, allows to obtain numerical solutions both in the case of the lifting line (Chapter 6) and the case of the lifting surface (Chapter 5). In the case of supersonic flow, the integral equations are solved analytically. For the three-dimensional wing (the lifting surface) we present in Chapter 8 a nice solution given by D. Homentcovschi in [8.16]. The integral equations describing the flow past oscillatory wings and profiles (chapter 10) have the same nature like the equations utilized in the case of steady flow but the kernels are more complicated. However for the sonic and supersonic flows these equations may be solved exactly by means of the Laplace transform, as it is shown in [10.17]. Chapter 9, devoted to the transonic motions, begins with a new asymptotic deduction of the equations of motion. The two and three-dimensional integral equations are obtained following the papers of the author and D. Homentcovschi. The theory of subsonic and supersonic flow past slender bodies (in Chapter 11) relies also on the fundamental solutions theory. In Chapter 2 one deduces the equations of the linear aerodynamics, on the basis of an asymptotic analysis, assuming that the small parameter depends on the thickness of the profile. In the classical aerodynamics this deduction is performed under the assumption that the unknowns and their derivatives have the same order of magnitude, but this fact cannot be *a priori* assumed. Then one calculates the fundamental solutions for the equation of the potential (paper [2.11]) and the fundamental solutions for the systems of equations of aerodynamics : the steady system[2.8], the oscillatory system [10.17], the unsteady system [2.6], [2.7]. On these solutions will rely the theories from the forthcoming chapters. The models we have already presented are the so called classical or linear models. They are suitable for the thin wings and thin profiles because they rely on the following assumptions: 1) one uses a linear boundary condition, 2) the boundary condition is imposed on the support of the wing (the segment $[-1,1]$ for the profile, the plan-form D for the three-dimensional wing), 3) the equations of motion are linearized. The development of the scientific computing allows us to develop more exact methods. Indeed we can give up to the first two assumptions using the boundary integral equations method (BIEM), also called the boundary element method (BEM), which was employed for the first time by Hess and Smith [7.9], [7.10]. The integral equations on the boundary are obtained imposing the exact boundary condition on the boundary of the wing. The integral equation is discretized using, for example, the collocation method. One obtains an algebraic system which is solved numerically. The linearization of the equations of motion is necessary only in the case of compressible fluids. The theory that we have developed is thus valid for every body in an incompressible fluid and for a thin body in a compressible fluid. Two chapters from this book, Chapter 4 for the 2d airfoil and Chapter 7 for the 3d airfoil are based on our papers (L. Dragoş and A. Dinu). The comparison between the known analytical results and the numerical results shows a very good agreement. In the Appendices we give some results concerning The Distributions Theory, The Singular Integral Equations Theory, The Principal Value and The Finite Part, Gauss-type Quadrature Formulas, etc.

In every work one finds, in a certain measure, both the achievements of the predecessors and of the researchers contemporaneous with the author. Among the people which have directly collaborated with me, I have to mention at first my professors Victor Vâlcovici and Caius Iacob, who introduced me in the field of aerodynamics. I also mention my younger colleagues Nicolae Marcov, Liviu Dinu, Dorel Homentcovschi, Adrian Carabineanu, Victor Ţigoiu, Vladimir Cardoş, Gabriela Marinoschi, Stelian Ion and Adrian Dinu. They were my students at the University of Bucharest, but I learned a lot from their papers. Some of them were my fellow - workers in the aerodynamics research, many of them stimulated me with their youth and their way of thinking in our seminars from the Faculty of Mathematics of the University of Bucharest. I am very grateful to all of them.

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