Preface

The notion of probability, and consequently the mathematical theory of probability, has in recent years become of interest to many scientists and engineers. There has been an increasing awareness that not "Will it work?" but "What is the probability that it will work?" is the proper question to ask about an apparatus. Similarly, in investigating the position in space of certain objects, "What is the probability that the object is in a given region?" is a more appropriate question than "Is the object in the given region?" As a result, the feeling is becoming widespread that a basic course in probability theory should be a part of the undergraduate training of all scientists, engineers, mathematicians, statisticians, and mathematics teachers.

A basic course in probability theory should serve two ends.

On the one hand, probability theory is a subject with great charm and intrinsic interest of its own, and an appreciation of the fact should be communicated to the student. Brief explanations of some of the ideas of probability theory are to be found scattered in many books written about many diverse subjects. The theory of probability thus presented sometimes appears confusing because it seems to be a collection of tricks, without an underlying unity. On the contrary, its concepts possess meanings of their own that do not depend on particular applications. Because of this fact, they provide *formal* analogies between real phenomena, which are themselves totally different but which in certain theoretical aspects can be treated similarly. For example, the factors affecting the length of the life of a man of a certain age and the factors affecting the time a light bulb will burn may be quite different, yet similar mathematical ideas may be used to describe both quantities.

On the other hand, a course in probability theory should serve as a background to many courses (such as statistics, statistical physics, industrial engineering, communication engineering, genetics, statistical psychology, and econometrics) in which probabilistic ideas and techniques are employed. Consequently, in the basic course in probability theory one should attempt to provide the student with a confident technique for solving probability problems. To solve these problems, there is no need to employ intuitive witchcraft. In this book it is shown how one may formulate probability problems in a mathematical manner so that they may be systematically attacked by routine methods. The basic step in this procedure is to express any event whose probability of occurrence is being sought as a set of sample descriptions, defined on the sample description space of the random phenomenon under consideration. In a similar spirit, the notion of random variable, together with the sometimes bewildering array of notions that must be introduced simultaneously, is presented in easy stages by first discussing the notion of numerical valued random phenomena.

This book is written as a textbook for a course in probability that can be adapted to the needs of students with diverse interests and backgrounds. In particular, it has been my aim to present the major ideas of modern probability theory without assuming that the reader knows the advanced mathematics necessary for a rigorous discussion.

The first six chapters constitute a one-quarter course in elementary probability theory at the sophomore or junior level. For the study of these chapters, the student need have had only one year of college calculus. Students with more mathematical background would also cover Chapters 7 and 8. The material in the first eight chapters (omitting the last section in each) can be conveniently covered in thirty-nine class hours by students with a good working knowledge of calculus. Many of the sections of the book can be read independently of one another without loss of continuity.

Chapters 9 and 10 are much less elementary in character than the first eight chapters. They constitute an introduction to the limit theorems of probability theory and to the role of characteristic functions in probability theory. These chapters provide careful and rigorous derivations of the law of large numbers and the central limit theorem and contain many new proofs.

In studying probability theory, the reader is exploring a way of thinking that is undoubtedly novel to him. Consequently, it is important that he have available a large number of interesting problems that at once illustrate and test his grasp of the theory. More than 160 examples, 120 theoretical exercises, and 480 exercises are contained in the text. The exercises are divided into two categories and are collected at the end of each section rather than at the end of the book or at the end of each chapter. The theoretical exercises extend the theory; they are stated in the form of assertions that the student is asked to prove. The nontheoretical exercises are numerical problems concerning concrete random phenomena and illustrate the variety of situations to which probability theory may be applied. The answers to odd-numbered exercises are given at the end of the book; the answers to even-numbered exercises are available in a separate booklet.

In choosing the notation I have adopted in this book, it has been my aim to achieve a symbolism that is self-explanatory and that can be read as if it were English. Thus the symbol $F_X(x)$ is defined as "the distribution function of the random variable X evaluated at the real number x." The terminology adopted agrees, I believe, with that used by most recent writers on probability theory.

The author of a textbook is indebted to almost everyone who has touched the field. I especially desire to express my intellectual indebtedness to the authors whose works are cited in the brief literature survey given in section 8 of Chapter 1.

To my colleagues at Stanford, and especially to Professors A. Bowker and S. Karlin, I owe a great personal debt for the constant encouragement they have given me and for the stimulating atmosphere they have provided. All have contributed much to my understanding of probability theory and statistics.

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