

A Geometrical Proof of the Existence of Heteroclinic Orbits for the Lorenz Minimum Hydrodynamic Equations

Dan Dobrovolschi

April 5, 2006

Abstract - We consider a simple three dimensional quadratic system which arises in several branches of mathematical physics. In geophysical fluid dynamics, this system was introduced in 1960 by E.N. Lorenz under the name of the minimum hydrodynamic equations (MHEs). The MHEs are the simplest spectral form of barotropic vorticity equation and describe qualitatively nonlinear barotropic phenomena of the atmosphere. Previous mathematical studies of MHEs were only of analytical and/or numerical nature. In this paper, by using a geometrical approach, we completely describe the phase portrait of the MHEs. As a by-product, we also obtain the existence of heteroclinic solutions. Our method is first to reduce by rescalings the initial system with three parameters to an equivalent system with no parameters. Next we determine the phase portrait of the reduced system by using two independent first integrals of it.

Key words and phrases : minimum hydrodynamic equations, phase portrait, heteroclinic orbits

Mathematics Subject Classification (2000) : 34A26, 86A10