Analele Universității București, Matematică Anul LVII(2008), pp. 247–258

Another proof for the continuity of the canonical projection from the shift space on the attractor of a certain infinite IFS

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Abstract - For A an arbitrary set and a fixed point z of A, let us consider the set $A' = A - \{z\}$. For a given p, the points of $l^p(A)$ are collections of real numbers indexed by points of A'. If E is the set of real numbers, then $x \in l^p(A)$ means $x = \{x_a\} \in E^{A'}$ such that $x_a = 0$ for all but countable many $a \in A'$ and $\sum x_a^p$ converges. The topology of $l^p(A)$ is induced from the metric $d(x, y) = (\sum_a |x_a - y_a|^p)^{\frac{1}{p}}$, where we think x_a as the a-th coordinate of x. For $p \in [1, \infty)$, let us consider the function $p_p : N(A) \to l^p(A)$ given by $p_p(\alpha) = (\alpha_b)_{b \in A'}$, where $\alpha = a_1 a_2 \cdots \in N(A)$ and $\alpha_b = \sum_{\substack{k \text{ with } a_k = b}} \frac{1}{2^k}$, if there exists k such that $a_k = b$ and $\alpha_b = 0$, otherwise, where N(A) is the Baire space (which is the topological product of countably many copies A_n of A). A. Mihail and R. Miculescu showed, using results concerning the shift

space for an infinite IFS, that p_p is continuous. In this paper we present a proof of this fact that avoids to use such type of arguments.

Key words and phrases : Lipscomb space, infinite iterated function system, attractor of an infinite IFS

Mathematics Subject Classification (2000) : Primary: 28A80; Secondary: 37C70, 54A20, 54B15