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Compact space-like submanifolds in a de Sitter space

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Abstract - In this paper, we investigate the n-dimensional ($n \geq 3$) compact space-like submanifolds M^n in a de Sitter space $S_p^{n+p}(c)$. Suppose that the normalized mean curvature vector field is parallel. We show that if $\|\nabla h\|^2 \geq n^2\|\nabla H\|^2$, then (1) if $H^2 < \frac{4(n-1)c}{m^2}$ on M^n, then M^n is totally umbilical; (2) if $H^2 = \frac{4(n-1)c}{m^2}$ on M^n, then M^n is totally umbilical; (3) if $H^2 > \frac{4(n-1)c}{m^2}$ on M^n and the squared norm of the second fundamental form $\|h\|^2$ satisfies $\|h\|^2 \leq nH^2 + (B_H^-(n,p,H))^2$ or $\|h\|^2 \geq nH^2 + (B_H^+(n,p,H))^2$ on M^n, then M^n is totally umbilical, where $m^2 = (n-2)^2p + 4(n-1)$ and $B_H^{\pm}(n,p,H)$ are the two real roots of a second-degree polynomial. As corollaries, we also obtain some general rigidity results.

Key words and phrases : space-like, submanifold, de Sitter space, totally umbilical.

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