

Singular Lagrange Spaces with (α, β) -metrics. Euler-Lagrange Equations

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Abstract - We study in this paper the Euler-Lagrange equations in the singular Lagrange space SL^n with (α, β) -metrics.

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1 Singular Lagrange space with an (α, β) -metric

The singular Lagrange space with an (α, β) -metric is a particular case of a singular Lagrange space which is studied in [13]. The Lagrange spaces with (α, β) -metrics are studied in [11]. The singular Finsler space was studied by T. Nagano ([9]) and the case of singular Riemannian space was studied by Gr. Moisil ([8]) and V. Oproiu ([12]).

In the present paper we define the notion of singular Lagrange space with (α, β) -metrics. We study the Euler-Lagrange equations and apply this theory to the extremal curves of these spaces.

Let (TM, τ, M) be the tangent bundle of a C^∞ -differentiable, real, n -dimensional manifold M . If (U, φ) is a local chart on M , then the coordinate of a point $u = (x, y) \in \tau^{-1}(U) \in TM$ will be denoted (x^i, y^i) . A differentiable Lagrangian on TM is a mapping

$$L : (x, y) \in TM \rightarrow L(x, y) \in \mathbf{R}, \quad \forall u = (x, y) \in TM$$

of class C^∞ on $\tilde{TM} \setminus \{0\}$ and continuous on the null section of the projection $\tau : TM \rightarrow M$. We consider the function defined on TM

$$\alpha(x, y) = \sqrt{\gamma_{ij}(x)y^i y^j}, \quad \beta(x, y) = A_i(x)y^i$$