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On Hypersurfaces in Spheres

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Abstract - We discuss some properties of minimal hypersurfaces in spheres with constant squared norm of the second fundamental form.
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1 Minimal hypersurfaces in spheres

Let $S^{n+1}(1)$ be a $(n+1)$-dimensional unit sphere and M^n a compact minimally immersed hypersurface in S^{n+1}. We denote by S the square of the length of h, the second fundamental form on M. It follows from the Gauss and Codazzi equations that the apparently extrinsic quantity S is, in fact, intrinsic and is given by

$$S = n(n-1) - R,$$

where R is the scalar curvature of M.

Chern proposed the following conjecture:

For a compact minimal hypersurface in the unit sphere S^{n+1}, with constant S, the values of S should be discrete.

For this conjecture, Simons proved that the first and the second value are 0 and n, respectively. He showed that if $0 \leq S \leq n$, everywhere, then $S \in \{0, n\}$. Clearly, M^n is contained in an equatorial sphere if $S = 0$. And when $S = n, M^n$ is indeed a piece of a product of spheres (Clifford torus), due to the works of Lawson, Chern , do Carmo and Kobayashi.

Perng and Terng made also a breakthrough and proved that if S is constant there exists a constant $\epsilon(n)$ such that if $n \leq S \leq n + \epsilon(n)$, then $S = n$ so that M is a Clifford torus.