Note on Geodesics in Distinguished Jet Framework

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Abstract - The basic types of curves on jet spaces endowed with non-linear connection (namely, the h-paths, v-paths, stationary curves and geodesics) are characterized. The main geometric objects and the paths of the framework are described and, in the special case when the vertical metric is independent of fiber coordinates, the first variation of energy is derived and the equations of geodesics are deduced. Several special classes of geodesics exemplify the developed theory.
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1 Preliminaries

The geometrized framework on first and higher-order osculating spaces was introduced and widely studied by Radu Miron and collaborators ([9]). As a complementary extension of the tangent (first-order osculating) framework, in the last decade, there was developed with significant results the geometric approach on first-order jet spaces ([13], [12], [3]).

Consider the first order jet bundle $\xi = (E = J^1(T, M), \pi, T \times M)$, of mappings $\varphi : T \to M$ - where T and M are $\mathcal{C}^\infty$ real differentiable manifolds ($\dim T = m$, $\dim M = n$). The local jet coordinates on E will be denoted by

$$(t^\alpha, x^i, y^A)_{(\alpha,i,A)\in I_*} \equiv (y^\mu)_{\mu\in I},$$

where the set of indices I splits as follows

$$I = I_h \cup I_v, \quad I_h = I_{h_1} \cup \mathrm{id}, \quad I_v = \overline{m+n+1, m+n+mn}$$

$$I_{h_1} = \overline{1, m}, \quad \mathrm{id} = \overline{m+1, m+n}, \quad I_* = I_{h_1} \times \mathrm{id} \times I_v.$$

and the indices implicitly take values as described below:

$$\alpha, \beta, \ldots \in I_{h_1}; \quad i, j, \ldots \in \mathrm{id}; \quad A, B, \ldots \in I_v; \quad \lambda, \mu, \ldots \in I.$$