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Some Aspects of the Educational and Scientific Work of Professor Kostake Teleman

Liviu NICOLESCU, Iulia Elena HIRICĂ

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We had the great honour to participate to the Scientific Session organized by the Faculty of Mathematics and Informatics, University of Bucharest, dedicated to Professor K. Teleman on his 70-th birthday.

We would like to express our appreciation for his permanent and enthusiastic work, during a half of a century, concerning education and mathematics.

Professor K. Teleman graduated University of Bucharest, Faculty of Mathematics in 1955. Immediately after that he became Assistant of the Department of Geometry and Topology (head of Department being Acad. Gh. Vrănceanu). He became Lecturer and since 1967 he is Associate professor. Prof. K. Teleman tought, year after year, to generations of students systematic knowledge of geometry, differential geometry, algebra, algebraic geometry and general topology, Donaldson-Witten theory, mechanics, mathematical physics, theory of relativity, Lie groups, Lie algebras, foundations of geometry, theory of curves and surfaces, history of mathematics.

Beyong these, one should remark his clear and deep perspective on the development of differential geometry, but also his example of correctitude and moral behaviour. Being former assistants of his lectures, the present authors can affirm the high scientific level of his lectures. During his fruitful teaching and research activity, all his students were permanently stimulated and helped in their studies and careers.

Through his important teaching activity, Prof. K. Teleman is among our great mathematicians Gh. Ţiţeica, Miron Nicolescu etc. We admired, during years, his great mathematical talent. His valuable monographs and lecture notes prove his deep mathematical culture. His clear and scientifical rigorous courses were very useful for students, master students, Ph.D. students and researchers in the field of differential geometry and topology.

In his monograph "Elements of topology and differential manifolds" (1964, Ed. D. P.) Prof. K. Teleman presents elements of general topology and most important methods used in algebraic topology and in geometry of differential manifolds. He also studies fibre bundles, simplicial polyhedra and cells complexes.

In the monograph "Methods and results in modern differential geometry" (1979, Ed. Şt. Enciclop. Bucureşti) are studied important aspects in differential geometry. Are presented local and global methods to solve systems of equations with partial derivatives. Having many applications in the equivalence problem of geometrical structures and in getting a wide class of invariants, these methods played an important role in the development of modern differential geometry. Using the methods of homological algebra and topology, introduced in the theory of the exterior differential systems, one can get global results.

Are presented applications in mechanics and in relativity, based on notions of differential geometry, useful for theories concerning the structure of the gravitational field and of the Universe.

It is studied, also, the theory of elliptic differential operators, which leads to developments in the field of differential geometry and topology.

His lecture notes improved the teaching in the field of geometry in our Faculty, in the last three decades of the last century.

We attended with pleasure his interesting lectures presented in "Gh. Ţiţeica" and "Gh. Vrănceanu" Seminars of the Department of Geometry and also his numerous communications at conferences organized by our University or other Universities. We noticed the importance accorded by him to the mathematical work of our great geometers Gh. Ţiţeica, Gh. Vrănceanu and J. Bolyai.

He was a student of Acad. Gh. Vrănceanu and he continued the research initiated by his professor. He started to publish original papers on differential geometry as a student in 1953. His fields of research in differential geometry are:

- theory of spaces with affine connection;
- symmetric Riemann spaces;
- quaternionic projective spaces;
- holonomy groups associated to an infinitesimal connection;
- osculating fibre bundles to a Riemann surface;
- motion group of Riemann spaces;
- eccentric anomalies in the theory of relativity;

- spaces with constant curvature;
- global differential geometry;
- algebraic topology;
- Bäcklund transforms;
- classification of elementary particles;
- conformal Lorentz geometry;
- quantification of relativistic Lagrangeans.

In algebraic topology he investigated the simultaneous generalization of homology and homotopy groups, spectral sequences and Casson handles.

Some of his works were joint, such as geometry courses for IX-X-th classes of highschool, written in 1979-1982.

One can have an idea of the dimension of his work by this simple enumeration of directions of research, in which he has remarkable contributions. His scientific work was published in Romania or abroad; some of his papers appeared in important reviews abroad. His articles are cited by many researchers, some of them being important geometers (Helgason, Greub, Halperin, Vanstone, Kobayashi, Nomizu etc).

It is quite difficult to present his whole work completely. So, we only underline concisely some of his major contributions, often cited and used by us.

1) Fourty years ago, during a discussion between Acad. Gh. Vrănceanu and Acad. Gr. Moisil was mentioned a natural method, developed afterwards by Acad. Gh. Vrănceanu. This method gives the possibility to associate to every finite dimensional real algebra a space with constant affine connection. The structure constants of the algebra were considered as coefficients of the connection. Conversely, the coefficients of every connection can be taken as structure constants, in certain basis, of a finite dimensional real algebra. In this way, one can obtain a correspondence between certain algebraic properties and geometric properties. So, having a space with constant affine connection A_n, the property of being locally Euclidean, is equivalent with the fact that the associated algebra is commutative and associative. Prof. K. Teleman gave a recurrent method, with respect to the dimension, of all commutative and associative finite dimensional algebras and hence of all locally Euclidean spaces with constant connection A_n. His result proves the equivalence between the following problems:

i) Find the finite dimensional commutative and associative algebras;

ii) Find the abelian, linear groups which are locally transitive in complex affine spaces;

iii) Find the abelian, linear groups which are locally transitive in complex projective spaces;

iv) Find the local Euclidean spaces with constant connection;

v) Find the projective Euclidean spaces with affine connection, having constant the associated Thomas connection.

2) In a paper published in 1956, Gh. Vrănceanu generalises the results of Levi-Civita, concerning Riemann spaces geodesically related. Gh. Vrănceanu proves that two non-trivial geodesically related Riemann spaces, one of them being irreducible, have homothetic metrics. This property was improved by Prof. K. Teleman in his paper " Ob adnoi teoreme Borelia-Lichnerowicza" (1958, Rev. Roum. de Math.). His result, cited by italians, russians and japanesse mathematicians proves that the curvature Riemann tensor, having a certain irreducibility property, determines the metric of the space, modulo a constant factor.

3) The complex projective spaces $V_{2n} = P^n(\mathbf{C})$ and quaternionic spaces $V_{4n} = P^n(\mathbf{H})$, introduced by Gh. Vrănceanu and K. Teleman (Bull. Şt., 7 (1955)) can be endowed with symmetric Riemann metrics, such that these spaces are considered as non-holonomic spaces of spheres S^{2n+1}, S^{4n+3}. This property is related to the fact that the spheres S^{2n+1}, S^{4n+3} are the total spaces of some fibre bundles, with basis $P^n(\mathbf{C})$, and $P^n(\mathbf{H})$. These are the Hopf fibre bundles.

In the paper " On a class of symmetric Riemann spaces" (Revue Roum. de Math., 2 (1957)) Prof. K. Teleman proves the following theorem:

Every closed symmetric Riemann space V_n can be defined as a non-holonomic subspace of a sphere, the metric of V_n being equal with the metric of the sphere induced on the non-holonomic subspace.

This article is cited and used, amongst others, by Helgason, Kobayashi and Nomizu.

4) Grassmann manifolds $M = G_p(\mathbf{E}^n)$ were studied by E. Cartan from a global Riemannian point of view. He proved that these spaces are symmetric.

Prof. K. Teleman extends Cartan's results in a paper published in 1958, cited by Kobayashi and Nomizu, underlining local properties of Grassmann manifolds M. It is given the metric of M, using two representations of M; the first of them is given in the space of automorphisms group and is a non- holonomic one. The second representation applies M topologically and isometrically in the Euclidean space $\mathbf{E}^{n^2}$.

5) In a paper published in 1963 (J.Math.Soc.Japan 15, 134-158), cited by Kobayashi and Nomizu, Prof. K. Teleman continues an article from 1954

and classifies Riemann spaces, with relatively many isometries. He used new and interesting techniques to study the isometry groups. He proved that if the isometry group G of a Riemann manifold V, with non constant curvature, has a linear irreducible subgroup as isotropy group of a point $x \in V$, then the dimension of G is at most $n + p^2$, where $n = dimV$ and $n = 2p$ or $n = 2p + 1$.

From a geometrical point of view, is studied the isometry group under which a Pfaff system is invariant. It is shown: *A group of isometries of the sphere S^{n-1}, under which a non-trivial Pfaff system is invariant, and which is irreducible as a linear group, has the dimension at most p^2, where $n = 2p$ or $n = 2p + 1$.*

6) In a paper cited and used amongst others by Greub, Halperin, Vanstone, Kobayashi, Nomizu and published in Annali di Mat. (62 (1963), 379-421), Prof. K. Teleman studies infinitesimal connections defined in differentiable fibre bundles of a given basis.

7) In 1973, the purpose of his research was to generalise and dualise a principle to get invariants of a group, principle introduced and used by Gh. Ţiţeica. Prof. K. Teleman obtained models with symmetry in the physics of elementary particles, using some principles of Gh. Ţiţeica. Some of the models were results of joint works with Prof. M. Teleman.

We would like to point out that, through his remarkable and fruitful scientifical and educational work, many students got interested in geometry and became valuable researchers. Some of his former students teach in our Faculty or in another Faculties in our country, some of them are researchers of the Mathematical Institute of Romanian Academy and others work in Universities abroad (U.S.A., France, Italy, Switzerland, Great Britain, Australia etc.).

It is hard to overestimate the number of scientists, all over the world, who have been initiated, permanently stimulated and helped, in their studies and careers, in research and in teaching, by Prof. K. Teleman.

Prof. K. Teleman always supported and encouraged scientific research. For his outstanding scientific and educational activity, we would like to express our gratitude, on the behalf of generations of mathematicians, all of them former students.

Prof. K. Teleman is considered our most important mathematician, working in the geometry area.

At this jubilee we have the opportunity to wish to you Prof. K. Teleman, good health, inspiration for your creative work and a warm " Happy birthday!"

List of publications of Professor Kostake Teleman

1950

1. *On polar conics to cubics* , (in Romanian) Gazeta Matematică

1953

2. *Les groupes transitifs de mouvement des espaces de Riemann* V_5, Stud.Cerc.Mat.,4

1954

3. *Sur les groupes maximums de mouvement des espaces de Riemann* V_n, Stud.cerc.Mat.,5

4. *Sur le groupe des rotations*, Bul.Şt.al Acad.Rom., 6

1955

5. *Les transformations qui laissent invariant le groupe orthogonal*, Bul. de Ştiinţe Mat.al Acad.Rom., 7

6. *Sur les espaces symétriques* V_5, Stud.Cerc.Mat.,6

7. *Sur certains espaces symétriques*, Stud.Cerc.Mat.,7

8. *Sur certains espaces symétriques*, Bul.Şt. al Acad.Rom., 7

1956

9. *Sur les espaces à connexion affine* A_1 *complexes*, Rev.Math.Pures et Appl., 2

1957

10. *Une classe de fonctions analytiques d'une surface de Riemann, généralisant les integrales abéliennes*, Stud. Cerc. Mat., 8

11. *Sur une classe d'espaces riemanniens symétriques*, Rev.Math.Pures et Appl., 2

12. *Sur les systemes mécaniques nonholonomes*, An.Univ."C.I.Parhon" Bucureşti, 6

1958

13. *Sur les variétés de Grassmann*, Bull.Math.Soc.Sci.Math., 2

14. *Ob adnoi teoreme Borelia- Lichnerowicza*, Rev.Roum.Math.Pures Appl, 3

1959

15. *Sur les structures homographiques d'une surface de Riemann,* Rev. Math. Pures et Appl., 4

16. *Sur les structures homographiques d'une surface de Riemann*, Comment.Math.Helv., 33

17. *Généralisation du groupe fondamental d'un espace topologique*, C. R. Acad. Sci. Paris, 248

18. *Généralisation du groupe fondamental d'une variété différentiable,* C.R.Acad.Sci.Paris, 248

1960

19. *Généralisation du groupe fondamental*, Ann.Sci.Ecole Norm.Sup., 77

20. *Sur les structures fibrées osculatrices d'une surface de Riemann*, Comment.Math.Helv., 34

1961

21. *Sur la classification des espaces fibrés*, C.R.Acad.Sci.Paris, 253

22. *Sur la structure du groupe* Γ *associé à une variété différentiable compacte*, C.R.Acad.Sci.Paris, 253

1962

23. *Sur la structure de certains groupes topologiques*, General Topology and its relations to Modern Analysis and Algebra, (Proc.Sympos.Prague, 1961), Academic Press

1963

24. *Généralisation simultanée des groupes d'homologie et des groupes d'homotopie*, Topologie, 2

25. *Sur les groupes des mouvements d'un espace de Riemann*, J. Math. Soc. Japan, 15

26. *Sur les connexions infinitésimales qu'on peut definir dans les structures fibrées différentiables de base donnée*, Ann.Math. Pura Appl., 4

1964

27. *Elements of topology and differential manifolds* , (in Romanian) Ed. Didact. şi Ped.

1965

28. *Euclidean geometry, non-Euclidean geometries, theory of relativity* (with Gh.Vrănceanu), (in Romanian) Ed.Ştiinţifică

29. *On a class of homogeneous Riemannian spaces*, (in Romanian) Stud. Cerc. Mat., 17

1966

30. *Sur les équations de Maurer-Cartan*, Rev.Roum.Math. Pures Appl., 11

31. *Recherches de géométrie différentielle en Roumanie*, (with Gh. Vrănceanu and Th. Hangan), Rev.Roum.Math.Pures Appl., 11

32. *On spaces with constant affine connection* , (in Romanian) Stud. Cerc. Mat., 18

1967

33. *Sur une théorie générale des connexions*, Bull.Math.Soc.Sci.Math., 10

34. *Sur le caractère de Chern d'un fibré vectoriel complexe différentiable*, Rev.Roum. Math. Pures Appl., 12

35. *Euclidean geometry, non-Euclidean geometry, theory of relativity* (with Gh.Vrănceanu), (in Romanian) Ed.Tehnică

36. *Elementi topologhii i diferenţiruemie mnogoobrazia*, (trad. N. M. Ostianu), Izd. Mir, Moscova

37. *Differential geometry on non-Hausdorff manifolds*, An.Univ.Buc., 16

38. *Elementary geometry from a modern point of view* , (with Gh. Vrănceanu and T. Hangan), (in Romanian) Ed.Soc.Şt.Mat., Bucureşti

1968

39. *On the principal series of unitary representations of the group* $SGL(n, \mathbf{C})$, Rev. Roum. Math. Pures Appl., 13

40. *Differential mappings from* $\mathbf{R^m}$ *to* $\mathbf{R^n}$, (in Romanian) An.Univ.Buc., 17

41. *Grundzuge der Topologie und differenzierbare Mannigfaltigkeiten*, (trad. Horst Antelmann), Deutscher Verlag der Wissenschaften, Berlin

1969

42. *Connections and bundles, I*, Nederl.Akad.Wetensch.Proc. Ser.A 72

43. *Connections and bundles, II*, Nederl.Akad.Wetensch.Proc.Ser. A 72

1972

44. *Un critère de classification des particules élémentaires,* (with Mihaela Teleman), C.R.Acad.Sci.Paris, 275

1973

45. *Elements of groups theory with applications in topology and physics,* (with Mihaela Teleman), (in Romanian) Ed.Ştiinţifică, Bucureşti

46. *Sur la classification des particules élémentaires* , (with Mihaela Teleman), C.R.Acad.Sci.Paris,

1974

47. *Global and local differential geometry* , (in Romanian) Ed. Tehnică, Bucureşti

1976

48. *Généralisation d'un théoreme de G.Tzitzeica*, Rev.Roum.Math.Pures Appl., 21

49. *Applications of topology in physics of elementary particles* (with Mihaela Teleman), (in Romanian) Ed.Ştiinţifică, Bucureşti

1977

50. *Généralisation des suites spectrales*, (with Ta Man), Enseignement Math.(2), 23

1979

51. *Methods and results in modern differential geometry,* (in Romanian) Ed.Ştiinţifică, Bucureşti

1982

52. *On some spaces with positive constant curvature* , (in Romanian) Stud.Cerc.Mat., 34

1983

53. *Sur une classe de faisceaux*, (with Oltin Dogaru), Rev. Roum. Math. Pures Appl., 28

54. *Faisceaux céllulaires*, Rev.Roum.Math.Pures Appl., 28

1986

55. *Sur un théoreme de Backlund*, (with Ana-Maria Teleman), St. Cerc. Mat., 38

56. *On Euclidean geometry*, (in Romanian) Gazeta Matematică,3, p. 65-66

57. *Logic and geometry* , (in Romanian) Gazeta Matematică, 6, 177-179; 9, 321-323

1989

58. *On Casson handles*, (with Andrei Teleman), St.Cerc.Mat., 41

1995

59. *Généralisation d'un theoreme de Backlund*, (with Ana-Maria Teleman, An.Univ.Bucureşti, 44

1996

60. *Conformal Lorentz geometry revisited*, J.Math.Phys., 37

1998

61. *Relativistic eccentric anomalies*, J.Math.Phys., 39

62. *Henri Poincaré,* Gazeta Matematică, 4, p. 169-170 (in Romanian)

1999

63. *A combined Backlund-Tzitzeica theorem*, (with Ana-Maria Teleman), An.Univ.Buc., 48

2000

64. *A classification theorem for connections*, Balkan J.Geom.Appl., 5

65. *Generalizing the fundamental group*, An.Univ.Buc., 49

66. *Dual De Sitter Spaces and Dirac Matrices*, An.Univ.Buc., Mat.-Inform., 49

2002

67. *Lorentz invariant Lagrangians*, J.Math.Phys., 43

Liviu Nicolescu, Iulia Elena Hirica
University of Bucharest, Faculty of Mathematics and Informatics
14 Academiei St., RO-010014, Bucharest, Romania
E-mail: lnicol@geometry.math.unibuc.ro
ihirica@geometry.math.unibuc.ro