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Variational Problems for a Witten Type Functional in Dimension 4

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Abstract - The aim of this article is to compute the first variation and the second variation formulas for a Witten type functional associated to a $Spin^G(4)$ -structure and to give some elementary properties. I have obtained more and also similar results in the $Spin^G(3)$-case. They can be found in [24], [25], [26] and [27]. The functional related to a $Spin^c(4)$ -structure ($G = S^1$) has been defined by Witten (see [29]) and it has been studied by a great number of mathematicians.

The definitions related to vector bundles and connections can be found in [4], [7], [8] (see p. 54-55 for fibre bundles associated with a given principal bundle and a left action of its structure group on a fixed manifold) and [28]. The general definitions of the Lie groups $Spin^G(n)$ ($n \in \mathbf{N}^*$), of $Spin^G(n)$-structures in $SO(n)$-principal bundles and of associated Seiberg-Witten monopole equations (for $SO(4)$-coframes bundles) have been introduced and studied in [19] (p. 509 for the definitions) and [21].

Key words and phrases : spinor, connection, curvature, Euler-Lagrange equations, critical point, Hessian
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1 Introduction

The Witten-type functionals we consider here are real valued functions defined on a product of two spaces, called the configuration space: one of connections in a vector (or principal) bundle over a compact, connected and oriented 4-dimensional Riemannian manifold (X, g) and the other of sections in a vector bundle over (X, g), all associated to a convenient Lie group G and a $Spin^G(4)$-structure in (X, g). We have used as a model the functional given by Witten in [W] in the abelian case ($G = S^1$).

It will be convenient for the theory to consider as well Sobolev completions of configuration space. Such functionals appear in physics like integrals