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On Generalized Spheres Associated to some Left Invariant Metrics

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Abstract - We follow our project, started in [4], to geometrize the level sets $S(g, \xi)$ ("generalized spheres") of the energy function f, associated to a given vector field ξ on a Riemannian manifold (M, g).

First, we give a new characterization for a critical point of f, in terms of a suitable Schouten connection associated to ξ.

We consider the case of some left invariant metrics on the generalized Heisenberg spaces and on the Damek-Ricci spaces; if ξ is the "position" vector field, then f has no critical points, so this generalized spheres are hypersurfaces. Finally, we generalize this result on arbitrary nilpotent groups, with respect to Malcev coordinates.

Key words and phrases : generalized spheres, energy function, generalized Heisenberg Lie groups, Damek-Ricci spaces, Schouten connection
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1 Introduction

Hypersurfaces defined by implicit equations in $\mathbf{R}^n$ are quite common in Differential Geometry. Among them, there are the well known constant level sets of an energy function, associated to a given vector field, via the Euclidean metric. The geometrization of the energy functions is important in applied sciences, where level sets depict distribution "charts" of some field forces.

More generally, consider a Riemannian manifold (M, g) and a fixed vector field ξ on M. The generalized sphere $S(g, \xi)$ of radious one is the set $S(g, \xi) = \{x \in M \mid g_x(\xi_x, \xi_x) = 1\}$ ([4]). In general, this set may have singular points, subject to classical conditions of Morse theory.