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An Application of Euler's Equation

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Abstract - This paper shows the simplest variational problem solved using finite differences method.

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Let $J[y]$ be a functional on a normed linear space $\mathcal{C}$.

Definition 1. *J has a local extremum for $y = \hat{y}$ if $J[y] - J[\hat{y}]$ does not change sign in a neighbourhood of $\hat{y}$.*

Theorem 1. *A necessary condition for a differentiable functional to have an extremum at $y = \hat{y}$ is that $\delta J\hat{y} = 0$, where δJ is the variation of $J[y]$.*

Proof. Let's suppose J has a local minum at $\hat{y}$. Then, by definition

$$\Delta J[y;h] = J[y+h] - J[y] \tag{1}$$

Then, this can be expressed as

$$\Delta J[y;h] = \delta J[\hat{y};h] + \epsilon \|h\| \tag{2}$$