On Hilbert and Riemann problems.
An alternative approach

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Abstract - Recall that the Hilbert (Riemann-Hilbert) boundary value problem was recently solved for arbitrary measurable coefficients and for arbitrary measurable boundary data in terms of nontangential limits and principal asymptotic values. Here it is developed a new approach making possible to obtain new results on tangential limits. It is shown that the spaces of the found solutions have the infinite dimension for prescribed collections of Jordan arcs terminating in almost every boundary point. Moreover, similar results are proved for the Riemann problem.

Key words and phrases : Hilbert and Riemann problems, analytic functions, limits along Jordan arcs, tangential limits, nonlinear problems.

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1. Introduction

The Hilbert (Riemann-Hilbert) boundary value problem, the Riemann and Poincaré boundary value problems are basic in the theory of analytic functions and they are closely interconnected, see for the history e.g. the monographs [3], [9] and [16], and also the last works [11]-[14].

Recall that the classical setting of the **Riemann problem** in a smooth Jordan domain D of the complex plane $\mathbb{C}$ was on finding analytic functions $f^+ : D \to \mathbb{C}$ and $f^- : \mathbb{C} \setminus \overline{D} \to \mathbb{C}$ that admit continuous extensions to ∂D and satisfy the boundary condition

$$f^+(\zeta) \;=\; A(\zeta) \cdot f^-(\zeta) \;+\; B(\zeta) \qquad \forall\, \zeta \in \partial D \qquad (1.1)$$

with prescribed Hölder continuous functions $A : \partial D \to \mathbb{C}$ and $B : \partial D \to \mathbb{C}$.

Recall also that the **Riemann problem with shift** in D was on finding such functions $f^+ : D \to \mathbb{C}$ and $f^- : \mathbb{C} \setminus \overline{D} \to \mathbb{C}$ satisfying the condition

$$f^+(\alpha(\zeta)) \;=\; A(\zeta) \cdot f^-(\zeta) \;+\; B(\zeta) \qquad \forall\, \zeta \in \partial D \qquad (1.2)$$

where $\alpha : \partial D \to \partial D$ was a one-to-one sense preserving correspondence having the non-vanishing Hölder continuous derivative with respect to the

natural parameter on ∂D. The function α is called a **shift function**. The special case $A \equiv 1$ gives the so–called **jump problem**.

The classical setting of the **Hilbert (Riemann-Hilbert) boundary value problem** was on finding analytic functions f in a domain $D \subset \mathbb{C}$ bounded by a rectifiable Jordan curve with the boundary condition

$$\lim_{z \to \zeta} \text{Re}\left\{\overline{\lambda(\zeta)} \cdot f(z)\right\} = \varphi(\zeta) \qquad \forall\, \zeta \in \partial D \qquad (1.3)$$

with functions λ and φ that are continuously differentiable with respect to the natural parameter s on ∂D and, moreover, $|\lambda| \neq 0$ everywhere on ∂D. Hence without loss of generality one can assume that $|\lambda| \equiv 1$ on ∂D.

It is clear that if we start to consider the Hilbert and Riemann problems with measurable boundary data, the requests on the existence of the limits at all points $\zeta \in \partial D$ and along all paths terminating in ζ lose any sense (as well as the conception of the index). Thus, the notion of solutions of the Hilbert and Riemann problems should be widened in this case. The nontangential limits were a suitable tool from the function theory of one complex variable, see e.g. [11]–[14]. Here it is proposed an alternative approach admitting tangential limits.

Given a Jordan curve C in $\mathbb{C}$, we say that a family of Jordan arcs $\{J_\zeta\}_{\zeta \in C}$ is of **class BS (of Bagemihl–Seidel class)**, cf. [1], 740–741, if all J_ζ lie in a ring $\mathfrak{R}$ generated by C and a Jordan curve C_* in $\mathbb{C}$, $C_* \cap C = \varnothing$, J_ζ is joining C_* and $\zeta \in C$, every $z \in \mathfrak{R}$ belongs to a single arc J_ζ, and for a sequence of mutually disjoint Jordan curves C_n in $\mathfrak{R}$ such that $C_n \to C$ as $n \to \infty$, $J_\zeta \cap C_n$ consists of a single point for each $\zeta \in C$ and $n = 1, 2, \ldots$.

In particular, a family of Jordan arcs $\{J_\zeta\}_{\zeta \in C}$ is of class BS if J_ζ is generated by an isotopy of C. For instance, every curvilinear ring $\mathfrak{R}$ one of whose boundary component is C can be mapped with a conformal mapping g onto a circular ring R and the inverse mapping $g^{-1} : R \to \mathfrak{R}$ maps radial lines in R onto suitable Jordan arcs J_ζ and centered circles in R onto Jordan curves giving the corresponding isotopy of C to other boundary component of $\mathfrak{R}$.

Now, if $\Omega \subset \mathbb{C}$ is an open set bounded by a finite collection of mutually disjoint Jordan curves, then we say that a family of Jordan arcs $\{J_\zeta\}_{\zeta \in \partial\Omega}$ is of class BS if its restriction to each component of $\partial\Omega$ is so.

2. On the Hilbert problem

Theorem 2.1. *Let D be a bounded domain in $\mathbb{C}$ whose boundary consists of a finite number of mutually disjoint rectifiable Jordan curves, and let $\lambda : \partial D \to \mathbb{C}$, $|\lambda(\zeta)| \equiv 1$, $\varphi : \partial D \to \mathbb{R}$ and $\psi : \partial D \to \mathbb{R}$ be measurable functions with respect to the natural parameter. Suppose that $\{\gamma_\zeta\}_{\zeta \in \partial D}$ is a family of Jordan arcs of class BS in D.*

Then there exist single-valued analytic functions $f : D \to \mathbb{C}$ such that

$$\lim_{z \to \zeta} \text{Re } \{\overline{\lambda(\zeta)} \cdot f(z)\} = \varphi(\zeta), \tag{2.1}$$

$$\lim_{z \to \zeta} \text{Im } \{\overline{\lambda(\zeta)} \cdot f(z)\} = \psi(\zeta) \tag{2.2}$$

along γ_ζ for a.e. $\zeta \in \partial D$ with respect to the natural parameter.

Remark 2.1. Thus, the space of all solutions f of the Hilbert problem (2.1) in the given sense has the infinite dimension for any prescribed φ, λ and $\{\gamma_\zeta\}_{\zeta \in D}$ because the space of all measurable functions $\psi : \partial D \to \mathbb{R}$ has the infinite dimension.

Proof.   Indeed, set $\Psi(\zeta) = \varphi(\zeta) + i\psi(\zeta)$ and $\Phi(\zeta) = \lambda(\zeta) \cdot \Psi(\zeta)$ for all $\zeta \in \partial D$. Then by Theorem 2 in [1] there is a single-valued analytic function f such that

$$\lim_{z \to \zeta} f(z) = \Phi(\zeta) \tag{2.3}$$

along γ_ζ for a.e. $\zeta \in \partial D$ with respect to the natural parameter. Then also

$$\lim_{z \to \zeta} \overline{\lambda(\zeta)} \cdot f(z) = \Psi(\zeta) \tag{2.4}$$

along γ_ζ for a.e. $\zeta \in \partial D$ with respect to the natural parameter.   □

Similar result can be formulated for arbitrary Jordan domains in terms of the harmonic measure.

Theorem 2.2. *Let D be a bounded domain in $\mathbb{C}$ whose boundary consists of a finite number of mutually disjoint Jordan curves, and let $\lambda : \partial D \to \mathbb{C}$, $|\lambda(\zeta)| \equiv 1$, $\varphi : \partial D \to \mathbb{R}$ and $\psi : \partial D \to \mathbb{R}$ be measurable functions with respect to the harmonic measure. Suppose that $\{\gamma_\zeta\}_{\zeta \in \partial D}$ is a family of Jordan arcs of class BS in D.*

Then there exist single-valued analytic functions $f : D \to \mathbb{C}$ such that

$$\lim_{z \to \zeta} \text{Re } \{\overline{\lambda(\zeta)} \cdot f(z)\} = \varphi(\zeta), \tag{2.5}$$

$$\lim_{z \to \zeta} \text{Im } \{\overline{\lambda(\zeta)} \cdot f(z)\} = \psi(\zeta) \tag{2.6}$$

along γ_ζ for a.e. $\zeta \in \partial D$ with respect to the harmonic measure.

Remark 2.2. Again, the space of all solutions f of the Riemann-Hilbert problem (2.5) in the given sense has the infinite dimension for any prescribed φ, λ and $\{\gamma_\zeta\}_{\zeta \in D}$ because the space of all functions $\psi : \partial D \to \mathbb{R}$ that are measurable with respect to the harmonic measure has the infinite dimension.

Proof. Theorem 2.2 is reduced to Theorem 2.1 in the following way.

First, there is a conformal mapping ω of D onto a circular domain $\mathbb{D}_*$ whose boundary consists of a finite number of circles and points, see e.g. Theorem V.6.2 in [4]. Note that $\mathbb{D}_*$ is not degenerate because isolated singularities of conformal mappings are removable that is due to the well-known Weierstrass theorem, see e.g. Theorem 1.2 in [2]. Applying in the case of need the inversion with respect to a boundary circle of $\mathbb{D}_*$, we may assume that $\mathbb{D}_*$ is bounded.

Remark that ω is extended to a homeomorphism ω_* of $\overline{D}$ onto $\overline{\mathbb{D}_*}$, see e.g. point (i) of Lemma 3.1 in [14]. Set $\Lambda = \lambda \circ \Omega$, $\Phi = \varphi \circ \Omega$ and $\Psi = \psi \circ \Omega$ where $\Omega : \partial\mathbb{D}_* \to \partial D$ is the restriction of $\Omega_* := \omega_*^{-1}$ to $\partial\mathbb{D}_*$. Let us show that these functions are measurable with respect to the natural parameter on $\partial\mathbb{D}_*$.

For this goal, note first of all that the sets of the harmonic measure zero are invariant under conformal mappings between multiply connected Jordan domains because a composition of a harmonic function with a conformal mapping is again a harmonic function. Moreover, a set $E \subset \partial\mathbb{D}_*$ has the harmonic measure zero if and only if it has the length zero, say in view of the integral representation of the harmonic measure through the Green function of the domain $\mathbb{D}_*$, see e.g. Section II.4 in [10].

Hence Ω and Ω^{-1} transform measurable sets into measurable sets because every measurable set is the union of a sigma-compact set and a set of measure zero, see e.g. Theorem III(6.6) in [15], and continuous mappings transform compact sets into compact sets. Thus, the functions λ, φ and ψ are measurable with respect to the harmonic measure on ∂D if and only if the functions Λ, Φ and Ψ are measurable with respect to the natural parameter on $\partial\mathbb{D}_*$.

Then by Theorem 2.1 there exist single-valued analytic functions $F : D \to \mathbb{C}$ such that

$$\lim_{w \to \xi} \mathrm{Re}\ \{\overline{\Lambda(\xi)} \cdot F(w)\} \ = \ \Phi(\xi)\ , \tag{2.7}$$

$$\lim_{w \to \xi} \mathrm{Im}\ \{\overline{\Lambda(\xi)} \cdot F(w)\} \ = \ \Psi(\xi) \tag{2.8}$$

along $\Gamma_\xi = \omega(\gamma_{\Omega(\xi)})$ for a.e. $\xi \in \partial\mathbb{D}_*$ with respect to the natural parameter.

Thus, by the construction the functions $f = F \circ \omega$ are the desired analytic functions $f : D \to \mathbb{C}$ satisfying the boundary conditions (2.5) and (2.6) along γ_ζ for a.e. $\zeta \in \partial D$ with respect to the harmonic measure. $\qquad\square$

Remark 2.3. Many investigations were devoted to the nonlinear Hilbert (Riemann-Hilbert) boundary value problems with conditions of the type

$$\Phi(\zeta, f(\zeta)) \ = \ 0 \qquad \forall\, \zeta \in \partial D\ , \tag{2.9}$$

see e.g. [6], [7] and [17]. It is natural also to weaken such conditions to

$$\Phi(\zeta, f(\zeta)) = 0 \qquad \text{for a.e.} \quad \zeta \in \partial D . \tag{2.10}$$

It is easy to see that the proposed approach makes possible also to reduce such problems to the algebraic and measurable solvability of the relation

$$\Phi(\zeta, v) = 0 \tag{2.11}$$

with respect to a complex-valued function $v(\zeta)$, cf. e.g. [5].

Through suitable modifications of Φ under the corresponding mappings of Jordan boundary curves onto the unit circle $\mathbb{S} = \{\zeta \in \mathbb{C} : |\zeta| = 1\}$, we may assume that ζ belongs to $\mathbb{S}$.

3. On the Riemann problem

Theorem 3.1. *Let D be a domain in $\overline{\mathbb{C}}$ whose boundary consists of a finite number of mutually disjoint rectifiable Jordan curves, $A : \partial D \to \mathbb{C}$ and $B : \partial D \to \mathbb{C}$ be measurable functions with respect to the natural parameter. Suppose that $\{\gamma_\zeta^+\}_{\zeta \in \partial D}$ and $\{\gamma_\zeta^-\}_{\zeta \in \partial D}$ are families of Jordan arcs of class BS in D and $\mathbb{C} \setminus \overline{D}$, correspondingly.*

Then there exist single-valued analytic functions $f^+ : D \to \mathbb{C}$ and $f^- : \overline{\mathbb{C}} \setminus \overline{D} \to \mathbb{C}$ that satisfy (1.1) for a.e. $\zeta \in \partial D$ with respect to the natural parameter where $f^+(\zeta)$ and $f^-(\zeta)$ are limits of $f^+(z)$ and $f^-(z)$ az $z \to \zeta$ along γ_ζ^+ and γ_ζ^-, correspondingly.

Furthermore, the space of all such couples (f^+, f^-) has the infinite dimension for every couple (A, B) and any collections γ_ζ^+ and γ_ζ^-, $\zeta \in \partial D$.

Theorem 3.1 is a special case of the following lemma on the generalized Riemann problem with shifts that can be useful for other goals, too.

Lemma 3.1. *Under the hypotheses of Theorem 3.1, let in addition $\alpha : \partial D \to \partial D$ be a homeomorphism keeping components of ∂D such that α and α^{-1} have the $(N)-$property of Lusin with respect to the natural parameter.*

Then there exist single-valued analytic functions $f^+ : D \to \mathbb{C}$ and $f^- : \overline{\mathbb{C}} \setminus \overline{D} \to \mathbb{C}$ that satisfy (1.2) for a.e. $\zeta \in \partial D$ with respect to the natural parameter where $f^+(\zeta)$ and $f^-(\zeta)$ are limits of $f^+(z)$ and $f^-(z)$ az $z \to \zeta$ along γ_ζ^+ and γ_ζ^-, correspondingly.

Furthermore, the space of all such couples (f^+, f^-) has the infinite dimension for every couple (A, B) and any collections γ_ζ^+ and γ_ζ^-, $\zeta \in \partial D$.

Proof. First, let D be bounded and let $g^- : \partial D \to \mathbb{C}$ be a measurable function. Note that the function

$$g^+ := \{A \cdot g^- + B\} \circ \alpha^{-1} \tag{3.1}$$

is measurable. Indeed, $E := \{A \cdot g^- + B\}^{-1}(\Omega)$ is a measurable subset of ∂D for every open set $\Omega \subseteq \mathbb{C}$ because the function $A \cdot g^- + B$ is measurable by the hypotheses. Hence the set E is the union of a sigma-compact set and a set of measure zero, see e.g. Theorem III(6.6) in [15]. However, continuous mappings transform compact sets into compact sets and, thus, $\alpha(E) = \alpha \circ \{A \cdot g^- + B\}^{-1}(\Omega) = (g^+)^{-1}(\Omega)$ is a measurable set, i.e. the function g^+ is really measurable.

Then by Theorem 2 in [1] there is a single-valued analytic function $f^+ : D \to \mathbb{C}$ such that

$$\lim_{z \to \xi} f^+(z) \; = \; g^+(\xi) \tag{3.2}$$

along γ_ξ^+ for a.e. $\xi \in \partial D$ with respect to the natural parameter. Note that $g^+(\alpha(\zeta))$ is determined by the given limit for a.e. $\zeta \in \partial D$ because α^{-1} also has the $(N)-$property of Lusin.

Note that $\overline{\mathbb{C}} \setminus \overline{D}$ consists of a finite number of (simply connected) Jordan domains $D_0, D_1, \ldots, D_m$ in the extended complex plane $\overline{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$. Let $\infty \in D_0$. Then again by Theorem 2 in [1] there exist single-valued analytic functions $f_l^- : D_l \to \mathbb{C}$, $l = 1, \ldots, m$, such that

$$\lim_{z \to \zeta} f_l^-(z) \; = \; g_l^-(\zeta) \; , \qquad g_l^- := g^-|_{\partial D_l} \; , \tag{3.3}$$

along γ_ζ^- for a.e. $\zeta \in \partial D_l$ with respect to the natural parameter.

Now, let S be a circle that contains D and let j be the inversion of $\overline{\mathbb{C}}$ with respect to S. Set

$$D_* = j(D_0), \quad g_* = \overline{g_0 \circ j}, \quad g_0^- := g^-|_{\partial D_0}, \quad \gamma_\xi^* = j\left(\gamma_{j(\xi)}^-\right), \quad \xi \in \partial D_* \; .$$

Then by Theorem 2 in [1] there is a single-valued analytic function $f_* : D_* \to \mathbb{C}$ such that

$$\lim_{w \to \xi} f_*(w) \; = \; g_*(\xi) \tag{3.4}$$

along γ_ξ^* for a.e. $\xi \in \partial D_*$ with respect to the natural parameter. Note that $f_0^- := \overline{g_* \circ j}$ is a single-valued analytic function in D_0 and by construction

$$\lim_{z \to \zeta} f_0^-(z) \; = \; g_0^-(\zeta) \; , \qquad g_0^- := g^-|_{\partial D_0} \; , \tag{3.5}$$

along γ_ζ^- for a.e. $\zeta \in \partial D_0$ with respect to the natural parameter.

Thus, the functions f_l^-, $l = 0, 1, \ldots, m$, form an analytic function $f^- : \overline{\mathbb{C}} \setminus \overline{D} \to \mathbb{C}$ satisfying (1.2) for a.e. $\zeta \in \partial D$ with respect to the natural parameter.

The space of all such couples (f^+, f^-) has the infinite dimension for every couple (A, B) and any collections γ_ζ^+ and γ_ζ^-, $\zeta \in \partial D$, in view of the above construction because of the space of all measurable functions $g^- : \partial D \to \mathbb{C}$ has the infinite dimension.

The case of unbounded D is reduced to the case of bounded D through the complex conjugation and the inversion of $\overline{\mathbb{C}}$ with respect to a circle S in some of the components of $\overline{\mathbb{C}} \setminus \overline{D}$ arguing as above. $\quad\square$

Remark 3.1. Some investigations were devoted also to the nonlinear Riemann problems with boundary conditions of the form

$$\Phi(\zeta, f^+(\zeta), f^-(\zeta)) = 0 \qquad \forall\, \zeta \in \partial D . \tag{3.6}$$

It is natural as above to weaken such conditions to the following

$$\Phi(\zeta, f^+(\zeta), f^-(\zeta)) = 0 \qquad \text{for a.e.} \quad \zeta \in \partial D . \tag{3.7}$$

It is easy to see that the proposed approach makes possible also to reduce such problems to the algebraic and measurable solvability of the relations

$$\Phi(\zeta, v, w) = 0 \tag{3.8}$$

with respect to complex-valued functions $v(\zeta)$ and $w(\zeta)$, cf. e.g. [5].

Through suitable modifications of Φ under the corresponding mappings of Jordan boundary curves onto the unit circle $\mathbb{S} = \{\zeta \in \mathbb{C} : |\zeta| = 1\}$, we may assume that ζ belongs to $\mathbb{S}$.

Example 3.1. For instance, correspondingly to the scheme given above, special nonlinear problems of the form

$$f^+(\zeta) = \varphi(\zeta, f^-(\zeta)) \qquad \text{for a.e.} \quad \zeta \in \partial \mathbb{D} \tag{3.9}$$

in the unit disk $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ are always solved if the function $\varphi : \mathbb{S} \times \mathbb{C} \to \mathbb{C}$ satisfies the **Caratheodory conditions**: $\varphi(\zeta, w)$ is continuous in the variable $w \in \mathbb{C}$ for a.e. $\zeta \in \mathbb{S}$ and it is measurable in the variable $\zeta \in \mathbb{S}$ for all $w \in \mathbb{C}$.

Furthermore, the spaces of solutions of such problems always have the infinite dimension. Indeed, the function $\varphi(\zeta, \psi(\zeta))$ is measurable in $\zeta \in \mathbb{S}$ for every measurable function $\psi : \mathbb{S} \to \mathbb{C}$ if the function φ satisfies the Caratheodory conditions, see e.g. Section 17.1 in [8], and the space of all measurable functions $\psi : \mathbb{S} \to \mathbb{C}$ has the infinite dimension.

Problems. Finally, it is necessary to point out the open problems on solvability of Hilbert and Riemann problems along any prescribed families of arcs but not only along families of the Bagemihl–Seidel class and, more generally, along any prescribed families of paths to a.e. boundary point.

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