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On a class of meromorphic functions of Janowski type

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Dedicated to Professor Cabiria Andreian Cazacu on her 85th Birthday

Abstract - In this paper, we introduce and investigate a new subclass of meromorphic functions of Janowski type, giving the coefficient bounds, a sufficient condition for a function to belong to the considered class and also a convolution property. The results presented provide generalizations of results given in earlier works.

Key words and phrases : meromorphic functions, subordination, coefficient bounds, convolution.

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1. Introduction

Let Σ denote the class of all meromorphic functions having the form

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the punctured unit disk

$$\mathcal{U}^* := \{z \in \mathbb{C} \,:\, 0 < |z| < 1\} =: \mathcal{U} \backslash \{0\}.$$

Also, let $\mathcal{A}$ be the class of analytic functions h in $\mathcal{U}$ given by

$$h(z) = z + \sum_{n=1}^{\infty} h_n z^n.$$

For a function $f \in \Sigma$ given by (1.1) and a function $g \in \Sigma$ having the form

$$g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} b_n z^n, \tag{1.2}$$

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the convolution (or Hadamard product) of f and g is defined by

$$(f * g)(z) \equiv f(z) * g(z) := \frac{1}{z} + \sum_{n=1}^{\infty} a_n b_n z^n.$$

If f and g are two functions analytic in $\mathcal{U}$, then f is said to be subordinate to g, written as $f \prec g$ or $f(z) \prec g(z)$, if there exists a Schwarz function ω (i.e. analytic in $\mathcal{U}$, with $\omega(0) = 0$ and $|\omega(z)| < 1$, $z \in \mathcal{U}$) such that $f(z) = g(\omega(z))$. In particular, if g is univalent in $\mathcal{U}$, then $f \prec g$ if and only if $f(0) = g(0)$ and $f(\mathcal{U}) \subset g(\mathcal{U})$.

A function $f \in \Sigma$ is said to be meromorphic starlike of order α $(0 \leq \alpha < 1)$ if it satisfies the inequality

$$-\Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha \quad (z \in \mathcal{U}^*).$$

The class of all functions meromorphic starlike of order α is denoted by $\Sigma^*(\alpha)$, with $\Sigma^* := \Sigma^*(0)$. Also, a function f is in the class $\mathcal{MC}$ of meromorphic close-to-convex functions if there exista a function $g \in \Sigma^*$ such that the following condition holds:

$$-\Re\left(\frac{zf'(z)}{g(z)}\right) > 0 \quad (z \in \mathcal{U}^*).$$

Gao and Zhou [1] introduced and studied the class $\mathcal{K}_s$ consisting of all the functions h in $\mathcal{A}$ such that

$$\Re\left(\frac{z^2h'(z)}{g(z)g(-z)}\right) < 0 \quad (z \in \mathcal{U}),$$

where g belongs to the class $\mathcal{S}^*(1/2)$ of analytic starlike functions of order $1/2$ (i.e. the class of functions $g \in \mathcal{A}$ which satisfy the inequality $\Re(zg'(z)/g(z)) > 1/2$, $z \in \mathcal{U}$). For several recently investigated classes of analytic functions related to the above mentioned class $\mathcal{K}_s$, we refer to [2], [3], [9], [11] or [13].

Wang et al. [12] considered the class $\mathcal{MK}$ of meromorphic functions f from Σ which satisfy the inequality

$$\Re\left(\frac{f'(z)}{g(z)g(-z)}\right) > 0 \quad (z \in \mathcal{U}^*),$$

where $g \in \Sigma^*(1/2)$. More recently, Sim and Kwon [7] investigated the class $\Sigma(A, B)$ of functions $f \in \Sigma$ with the property

$$\frac{f'(z)}{g(z)g(-z)} \prec \frac{1 + Az}{1 + Bz},$$

with $-1 \le B < A \le 1$ and $g \in \Sigma^*(1/2)$, and in a similar manner, Soni and Kant [8] introduced and discussed $\mathcal{MK}(t,A,B)$ consisting of functions $f \in \Sigma$ for which the following subordination holds:

$$\frac{-f'(z)}{tg(z)g(tz)} \prec \frac{1+Az}{1+Bz},$$

where $0 < |t| \le 1$ and A, B and g are defined as above.

Motivated by the aforementioned works, we introduce and investigate the following subclass of meromorphic functions:

Definition 1.1. *Let $-1 \le B < A \le 1$ and $0 \le \alpha \le 1$. A function $f \in \Sigma$ of the form* (1.1) *is said to be in the class $\Sigma(A,B;\alpha)$ if there exists $g \in \Sigma^*(1/2)$ such that the following subordination is satisfied:*

$$\frac{(1-2\alpha)f'(z) - \alpha z f''(z)}{g(z)g(-z)} \prec \frac{1+Az}{1+Bz}. \tag{1.3}$$

The class $\Sigma(A,B;\alpha)$ provides a generalization of the classes studied by Wang, Sun and Xu [12] (the case $\alpha = 0$, $A = -1$ and $B = 1$) and Sim and Kwon [7] (the case $\alpha = 0$). Recently Shi, Yi and Wang [10] obtained results on the class $\Sigma(A,B;\alpha)$ when $\alpha \le 0$.

Remark 1.1. If $-1 \le B_2 \le B_1 < A_1 \le A_2 \le 1$, then $\Sigma(A_1,B_1;\alpha) \subset \Sigma(A_2,B_2;\alpha)$. To prove this, let $f \in \Sigma(A_1,B_1;\alpha)$. Then

$$\frac{(1-2\alpha)f'(z) - \alpha z f''(z)}{g(z)g(-z)} \prec \frac{1+A_1 z}{1+B_1 z}.$$

But since $-1 \le B_2 \le B_1 < A_1 \le A_2 \le 1$, the following subordination is true:

$$\frac{1+A_1 z}{1+B_1 z} \prec \frac{1+A_2 z}{1+B_2 z}. \tag{1.4}$$

Indeed, when $-1 < B_2 \le B_1$, the images of $\mathcal{U}$ under these two functions are two circles orthogonal on the real axis and also we have that

$$\begin{aligned}\min_{z\in\partial\mathcal{U}} \Re\frac{1+A_2 z}{1+B_2 z} &= \frac{1-A_2}{1-B_2} \le \min_{z\in\partial\mathcal{U}} \Re\frac{1+A_1 z}{1+B_1 z} = \frac{1-A_1}{1-B_1}\\ \le \max_{z\in\partial\mathcal{U}} \Re\frac{1+A_1 z}{1+B_1 z} &= \frac{1+A_1}{1+B_1} \le \max_{z\in\partial\mathcal{U}} \Re\frac{1+A_2 z}{1+B_2 z} = \frac{1+A_2}{1+B_2},\end{aligned}$$

which shows that the image of $\mathcal{U}$ under $(1+A_1 z)/(1+B_1 z)$ is included in the image of $\mathcal{U}$ under $(1+A_2 z)/(1+B_2 z)$, and so the subordination (1.4) holds. A similar argument shows the subordination is also true when $-1 = B_1 = B_2$ or $-1 = B_1 < B_2$. It therefore follows that $f \in \Sigma(A_2,B_2;\alpha)$.

In our investigation of the class $\Sigma(A,B;\alpha)$ we shall need the following lemmas:

Lemma 1.1. (see [12]) *Let* $g \in \Sigma^*(1/2)$. *Then*

$$-zg(z)g(-z) \in \Sigma^*.$$

Lemma 1.2. (see [12]) *Let*

$$g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} b_n z^n \in \Sigma^*(1/2).$$

Then

$$|B_{2n-1}| \leq \frac{1}{n} \quad (n \in \mathbb{N} = \{1, 2, \ldots\}),$$

where

$$B_{2n-1} = 2b_{2n-1} + 2b_1 b_{2n-3} - 2b_2 b_{2n-4} + \cdots + (-1)^{n-1} b_{n-2} b_n + (-1)^n b_{n-1}^2. \tag{1.5}$$

Lemma 1.3. (see [4]) *Let* $G \in \Sigma^*$. *Then for* $|z| = r$, $0 < r < 1$, *we have*

$$\frac{(1-r)^2}{r} \leq |G(z)| \leq \frac{(1+r)^2}{r}.$$

We need at this moment to give the definition of prestarlike functions. For $\gamma < 1$, the class $\mathcal{R}(\gamma)$ of prestarlike functions of order γ consists of all the normalized analytic functions f which satisfy the subordonation

$$f(z) * \frac{z}{(1-z)^{2-2\gamma}} \in \mathcal{S}^*(\gamma).$$

The class $\mathcal{R}(1)$ is formed with analytic normalized functions f for which the inequality $\Re(f(z)/z) > 1/2$ holds true.

Lemma 1.4. (see [6]) *Let* $\gamma \leq 1$, $f \in \mathcal{R}(\gamma)$ *and* $g \in \mathcal{S}^*(\gamma)$. *Then*

$$\frac{f * gF}{f * g}(\mathcal{U}) \subset \overline{\text{co}}(F(\mathcal{U})),$$

where F *is an analytic function in* $\mathcal{U}$ *and* $\overline{\text{co}}(F(\mathcal{U}))$ *denotes the closed convex hull of* $F(\mathcal{U})$.

Lemma 1.5. (see [5]) *Let*

$$h(z) = 1 + \sum_{n=1}^{\infty} h_n z^n \quad \text{and} \quad k(z) = 1 + \sum_{n=1}^{\infty} k_n z^n$$

be two analytic functions in $\mathcal{U}$. *If* k *is convex and* $h \prec k$, *then*

$$|h_n| \leq |k_1| \quad (n \in \mathbb{N}).$$

2. Main results

The following result gives a sufficient condition for a function to belong to the investigated class $\Sigma(A, B; \alpha)$.

Theorem 2.1. *Let* $-1 \leq B < A \leq 1$, $0 \leq \alpha \leq 1$ *and*

$$g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} b_n z^n.$$

If $f \in \Sigma$ *given by* (1.1) *satisfies the condition*

$$\sum_{n=1}^{\infty} [(1+|B|)|1-\alpha-\alpha n||a_n|n + (1+|A|)|B_{2n-1}|] < A - B \tag{2.1}$$

where the coefficients B_{2n-1} *are given by* (1.5)*, then* $f \in \Sigma(A, B; \alpha)$.

Proof. To prove $f \in \Sigma(A, B; \alpha)$ it suffices to show that

$$\left|\frac{(1-2\alpha)f'(z) - \alpha z f''(z)}{g(z)g(-z)} - 1\right| < \left|A - B\frac{(1-2\alpha)f'(z) - \alpha z f''(z)}{g(z)g(-z)}\right|. \tag{2.2}$$

Let

$$G(z) = -zg(z)g(-z). \tag{2.3}$$

It is obvious that $G(-z) = -G(z)$ and by Lemma 1.1 we deduce that G is a meromorphic odd starlike function. Therefore, G has the form

$$G(z) = \frac{1}{z} + \sum_{n=1}^{\infty} B_{2n-1} z^{2n-1}, \tag{2.4}$$

where B_{2n-1} is given by (1.5).

On taking

$$\begin{aligned}
\Delta &= |(1-2\alpha)zf'(z) - \alpha z^2 f''(z) + G(z))| \\
&\quad -|AG(z) + B[(1-2\alpha)zf'(z) - \alpha z^2 f''(z)]| \\
&= \left|\sum_{n=1}^{\infty}(1-\alpha-\alpha n)na_n z^n + \sum_{n=1}^{\infty} B_{2n-1} z^{2n-1}\right| \\
&\quad - \left|(A-B)\frac{1}{z} + \sum_{n=1}^{\infty} AB_{2n-1} z^{2n-1} + \sum_{n=1}^{\infty} B(1-\alpha-\alpha n)na_n z^n\right|,
\end{aligned}$$

by (2.1), we obtain the inequalities

$$\begin{aligned}
\Delta &\le \sum_{n=1}^{\infty}|(1-\alpha-\alpha n)a_n|n|z|^n+\sum_{n=1}^{\infty}|B_{2n-1}||z|^{2n-1} \\
&\quad -\left((A-B)\frac{1}{z}-\sum_{n=1}^{\infty}|AB_{2n-1}||z|^{2n-1}-\sum_{n=1}^{\infty}|B(1-\alpha-\alpha n)a_n|n|z|^n\right) \\
&= -(A-B)\frac{1}{z}+\sum_{n=1}^{\infty}(1+|B|)|(1-\alpha-\alpha n)a_n|n|z|^n \\
&\quad +\sum_{n=1}^{\infty}(1+|A|)|B_{2n-1}||z|^{2n-1} \\
&< -(A-B)+\sum_{n=1}^{\infty}(1+|B|)|(1-\alpha-\alpha n)a_n|n+\sum_{n=1}^{\infty}(1+|A|)|B_{2n-1}| \\
&\le 0.
\end{aligned}$$

Thus, because $\Delta < 0$, relation (2.2) holds true and we conclude that $f \in \Sigma(A,B;\alpha)$. □

We next determine the coefficient estimates for functions in $\Sigma(A,B;\alpha)$.

Theorem 2.2. *Let* $-1 \le B < A \le 1$, $0 \le \alpha \le 1$ *and* $f \in \Sigma(A,B;\alpha)$ *be given by* (1.1). *Then*

$$|a_1| \le 1,$$

$$|a_{2n}| \le \frac{A-B}{2n|1-(2n+1)\alpha|}\left(1+\sum_{k=1}^{n-1}\frac{1}{k}\right) \quad (n \in \mathbb{N}) \tag{2.5}$$

and

$$|a_{2n+1}| \le \frac{A-B}{(2n+1)|1-(2n+2)\alpha|}\left(1+\sum_{k=1}^{n}\frac{1}{k}\right) \quad (n \in \mathbb{N}). \tag{2.6}$$

Proof. Since $f \in \Sigma(A,B;\alpha)$, we know that

$$-\frac{(1-2\alpha)zf'(z)-\alpha z^2 f''(z)}{G(z)} \prec \frac{1+Az}{1+Bz},$$

where $G(z) = -zg(z)g(-z)$ is given by (2.4). Setting

$$q(z) = -\frac{(1-2\alpha)zf'(z)-\alpha z^2 f''(z)}{G(z)}, \tag{2.7}$$

it follows that

$$q(z) = 1 + d_1 z + d_2 z^2 + \cdots \quad \text{and} \quad q(z) \prec \frac{1+Az}{1+Bz}.$$

Moreover, by Lemma 1.5, we remark that

$$|d_n| \le A - B \quad (n \in \mathbb{N}). \tag{2.8}$$

Further, equation (2.7) gives

$$\begin{aligned} &(1 + d_1 z + d_2 z^2 + \cdots)\left(\frac{1}{z} + B_1 z + B_3 z^3 + \cdots\right) \\ = \;& \frac{1}{z} - (1-2\alpha)a_1 z - 2(1-3\alpha)a_2 z^2 - \cdots \\ - \;& 2n[1-(2n+1)\alpha]a_{2n}z^{2n} - (2n+1)[1-(2n+2)\alpha]a_{2n+1}z^{2n+1} - \cdots, \end{aligned}$$

from which we obtain $d_1 = 0$,

$$-2n[1-(2n+1)\alpha]a_{2n} = d_3 B_{2n-3} + \cdots + d_{2n-1}B_1 + d_{2n+1} \quad (n \in \mathbb{N})$$

and

$$-(2n+1)[1-(2n+2)\alpha]a_{2n+1} = d_2 B_{2n-1} + d_4 B_{2n-3} + \cdots + d_{2n}B_1 + d_{2n+2} \; (n \in \mathbb{N}).$$

From (2.8) and also considering Lemma 1.2, it results that

$$2n|1-(2n+1)\alpha||a_{2n}| \le (A-B)\left(\frac{1}{n-1} + \cdots + \frac{1}{2} + 1 + 1\right) \tag{2.9}$$

and

$$(2n+1)|1-(2n+2)\alpha||a_{2n+1}| \le (A-B)\left(\frac{1}{n} + \frac{1}{n-1} + \cdots + \frac{1}{2} + 1 + 1\right). \tag{2.10}$$

The conclusion in (2.5) and (2.6) follows now from (2.9) and (2.10), whereas the estimation $|a_1| \le 1$ is true for any meromorphic function f univalent in $\mathcal{U}^*$. □

Theorem 2.3. *If $-1 \le B < A \le 1$, $0 \le \alpha \le 1$ and $f \in \Sigma(A, B; \alpha)$ then for $|z| = r$, $0 < r < 1$, the following inequalities hold:*

$$\frac{(1-r)^2}{r^2}\,\frac{1-Ar}{1-Br} \le |(1-2\alpha)f'(z) - \alpha z f'(z)| \le \frac{(1+r)^2}{r^2}\,\frac{1+Ar}{1+Br}. \tag{2.11}$$

Proof. Suppose $f \in \Sigma(A, B; \alpha)$ and let

$$q(z) = -\frac{(1-2\alpha)zf'(z) - \alpha z^2 f''(z)}{G(z)},$$

where G given in (2.3) is a meromorphically starlike function. Since

$$q(z) \prec \frac{1+Az}{1+Bz},$$

by the subordination principle we obtain for $|z| = r$, $0 < r < 1$, that

$$\frac{1 - Ar}{1 - Br} \leq |q(z)| \leq \frac{1 + Ar}{1 + Br}.$$

Making use of Lemma 1.3, we readily obtain the desired inequalities, as asserted in (2.11). □

We provide next a convolution property of functions from the class $\Sigma(A, B; \alpha)$ considered.

Theorem 2.4. *Let $-1 \leq B < A \leq 1$, $0 \leq \alpha \leq 1$, $\gamma \leq 1$ and $f \in \Sigma(A, B; \alpha)$ such that the corresponding function $g \in \Sigma^*(1/2)$ satisfies the condition*

$$-\Re\left(\frac{zg'(z)}{g(z)}\right) < \frac{3}{2} - \frac{1}{2}\gamma \quad (z \in \mathcal{U}). \tag{2.12}$$

If $\phi \in \Sigma$ with $z^2\phi(z) \in \mathcal{R}(\gamma)$, then $\phi * f \in \Sigma(A, B; \alpha)$.

Proof. Let $f \in \Sigma(A, B; \alpha)$ and let G and q be given by

$$G(z) = -zg(z)g(-z), \quad q(z) = -\frac{(1 - 2\alpha)zf'(z) - \alpha z^2 f''(z)}{G(z)}.$$

As shown in the proof of Theorem 2.1, G is an odd meromorphic starlike function. Further, since

$$\frac{zG'(z)}{G(z)} = 1 + \frac{zg'(z)}{g(z)} - \frac{zg'(-z)}{g(-z)},$$

inequality (2.12) implies

$$-\Re\left(\frac{zG'(z)}{G(z)}\right) < 2 - \gamma \quad (z \in \mathcal{U}),$$

from which we obtain $z^2G(z) \in \mathcal{S}^*(\gamma)$.

Define $\varphi(z) = (\phi * G)(z)$ and $k(z) = \sqrt{z\varphi(z)}/z$. We observe that φ is also an odd and starlike meromorphic function. In order to prove this, let $F(z) = -zG'(z)/G(z)$. Since

$$\begin{aligned} -\frac{z(\phi * G)'(z)}{(\phi * G)(z)} &= \frac{(\phi * (-zG'))(z)}{(\phi * G)(z)} = \frac{\phi(z) * G(z)F(z)}{(\phi * G)(z)} \\ &= \frac{z^2\phi(z) * z^2G(z)F(z)}{z^2\phi(z) * z^2G(z)}, \end{aligned}$$

with $z^2G(z) \in \mathcal{S}^*(\gamma)$ and $z^2\phi(z) \in \mathcal{R}(\gamma)$, we deduce, by Lemma 1.4, that

$$-\frac{z(\phi * G)'(z)}{(\phi * G)(z)} \in \overline{\text{co}}(F(\mathcal{U})).$$

Because G is starlike meromorphic, we have $\Re F(z) > 0$, and so the above relation yields $\phi * G$ is indeed also starlike. As a consequence, $k \in \Sigma^*(1/2)$ and $\varphi(z) = -zk(z)k(-z)$. Moreover, we have

$$\begin{aligned} & \frac{(1-2\alpha)(\phi * f)'(z) - \alpha z(\phi * f)''(z)}{k(z)k(-z)} \\ = \quad & \frac{\phi * [-(1-2\alpha)zf'(z) + \alpha z^2 f''(z)]}{\varphi(z)} = \frac{\phi(z) * q(z)G(z)}{(\phi * G)(z)} \\ = \quad & \frac{z^2\phi(z) * q(z)z^2G(z)}{z^2\phi(z) * z^2G(z)}. \end{aligned}$$

By applying Lemma 1.4 once again we deduce that

$$\frac{z^2\phi(z) * q(z)z^2G(z)}{z^2\phi(z) * z^2G(z)} \in \overline{\text{co}}(q(\mathcal{U})). \tag{2.13}$$

But since $f \in \Sigma(A, B; \alpha)$, we have

$$q(z) \prec \frac{1 + Az}{1 + Bz}. \tag{2.14}$$

The function $(1+Az)/(1+Bz)$ is convex and therefore equations (2.13) and (2.14) yield

$$\frac{z^2\phi(z) * q(z)z^2G(z)}{z^2\phi(z) * z^2G(z)} \prec \frac{1 + Az}{1 + Bz},$$

which is equivalent to

$$\frac{(1-2\alpha)(\phi * f)'(z) - \alpha z^2(\phi * f)''(z)}{k(z)k(-z)} \prec \frac{1 + Az}{1 + Bz}.$$

Thus $\phi * f \in \Sigma(A, B; \alpha)$ and so the proof is complete. □

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