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DECENTRALIZED SUBOPTIMAL LQG CONTROL OF A PLATOON OF VEHICLES

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Abstract
In this paper the stochastic Inclusion Principle is applied to decentralized LQG suboptimal longitudinal control design of a platoon of automotive vehicles. Starting from a stochastic linearized platoon state model, input/state overlapping subsystems are defined and extracted after an adequate expansion. An algorithm for approximate LQG optimization of these subsystems is developed in accordance with their hierarchical LBT (lower-block-triangular) structure. Vehicle controllers obtained after contraction, which leaves local Kalman filters uncontracted, provide high performance tracking and noise immunity.

***Keywords**: Platoon of vehicles, Overlapping decompositions, LQG optimal control, Decentralized control*

1. INTRODUCTION

The problem of design of automated highway systems (AHS) has attracted a considerable attention among researchers, e.g. [2, 8]. AHS control architecture proposed in [8, 2, 18] is based on the introduction of a notion of platoons, groups of vehicles following the leading vehicles with small intra-platoon separation. Control of platoons has been studied from different viewpoints [9,7,16]. Main theoretical contributions are related to the stability problem [7, 16]. It has been shown that an efficient decentralized control law can be formulated when each vehicle is applied with data representing its acceleration, velocity, distance to the preceding vehicle, velocity and acceleration of the preceding vehicle, as well as velocity and acceleration references. [9]. However, tuning of the local regulator parameters has been based on arguments related to relative stability, without taking into account optimality in any predefined sense, structural and signal uncertainties and possibilities to improve the performance by introducing dynamics into the regulator. In [13, 14] a systematic procedure for the design of decentralized overlapping platoon controller on the basis of LQ optimization has been described.

In this paper a generalization of the approach in [13, 14] to the stochastic case is presented. Namely, the Stochastic Inclusion Principle [11, 12] is applied to the design of decentralized LQG suboptimal longitudinal control of a platoon of vehicles, taking into account uncertainty resulting from the influence of the environment and measuring devices. The first part of the paper contains the results related to platoon modeling, formulated in accordance with [8, 18,2,9,14], taking into account stochastic disturbances and measurement noise. A linearized stochastic state model for a string of moving vehicles is derived on the basis of [3, 9, 13]. Each vehicle is described by a state model, with accelerations, velocities and distances to preceding vehicles as state variables. In the second part, an outline of the theory of the Stochastic Inclusion Principle is presented. It is shown that a suitable expansion of the obtained platoon model which possesses the overlapping structure enables formal extraction of “subsystems” for which local quadratic performance indices can be formulated. Having in mind both the subsystem model structure and the available data set [9], an optimization technique resembling to the methodology for deriving LQ suboptimal control for systems with the hierarchical LBT structure proposed in [6, 10, 14], is developed and presented in the third part of the paper. Each subsystem controller contains a specific Kalman type estimator, together with the corresponding state feedback gain. Contraction to the original space provides a decentralized controller for the whole platoon, leaving all local state estimators uncontacted. Experimental results are given in order to illustrate main properties of the proposed methodology.

2. MODEL FORMULATION

It will be adopted in this paper that i-th automotive vehicle in a close formation platoon consisting of n vehicles can be represented by the following dynamic model:

$$\begin{aligned} &\dot{d}_i = v_{i-1} - v_i, \quad \dot{v}_i = a_i \\ &a_i = f_a^i(y_i - k_1^i v_i^2 - k_2^i - e_i), \quad \dot{y}_i = f_j^i(\alpha_i(u_i - y_i)), \end{aligned} \tag{1}$$

where $d_i = x_{i-1} - x_i$ is the distance between two consecutive vehicles, x_{i-1} and x_i represent their positions, v_i, a_i and $\dot{y}$ are the velocity, acceleration and jerk, respectively, $f_a^i(.)$ and $f_j^i(.)$ are static nonlinearities of saturation type, α_i represents the inverse time-constant of the basic vehicle dynamics, k_1^i and k_2^i constants defining rolling resistance, u_i is the corresponding control input, while e_i represents the white random noise force input with variance r_i^e, resulting from wind gusts and road roughness. A slightly modified version of (1) is taken in [9, 13] as the basic model of individual vehicles in a platoon. There are several possibilities for constructing linearized models in the state-space form depending on the choice of state variables, e.g. [3, 9, 8, 13, 14]. A convenient form follows directly from (1). Supposing for the sake of simplicity that n=3 and that all the vehicles have the same models, we obtain

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} A_v & 0 & 0 \\ A_d & A_v & 0 \\ 0 & A_d & A_v \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} B_v & 0 & 0 \\ 0 & B_v & 0 \\ 0 & 0 & B_v \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} G_e & 0 & 0 \\ 0 & G_e & 0 \\ 0 & 0 & G_e \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}, \qquad (2)$$

where $X_i^T = [d_i \quad v_i \quad a_i]$ ($x_0 = 0$ in d_1) and

$$A_v = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & +1 \\ 0 & 0 & -\alpha \end{bmatrix}, \quad A_d = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B_v^T = [0 \quad 0 \quad \alpha], \quad G_e^T = [0 \quad 1 \quad 0].$$

Control design for (2) can, obviously, be based on various methodologies. However, any attempt to formulate a globally optimal control law for the entire platoon is faced with the problem that control of each vehicle depends, in general, on the states of all the remaining vehicles. Permissible control strategies should essentially be decentralized, having in mind the supposed information structure [9], i.e. the local control u_i is to be calculated on basis of the noise measurements of the local vehicle state variables $\{d_i \quad v_i \quad a_i\}$, together with the noisy information about the velocity and acceleration of the preceding vehicle $\{v_{i-1} \quad a_{i-1}\}$, which is assumed to be transmitted by appropriate communication channels. Each vehicle is also supplied with the information about the spacing, velocity and acceleration reference command $\{d_r \quad v_r \quad a_r\}$. The theory of large scale systems abounds with methodologies for both decentralized design of complex control structures and decentralized design of completely decentralized control structures, e.g. [17, 13, 14]. One of elegant and powerful methodologies is based on the Stochastic Inclusion Principle [11,12].

3. STOCHASTIC INCLUSION PRINCIPLE

3.1 GENERAL ASPECTS

Consider a pair $(\mathbf{S}, \tilde{\mathbf{S}})$ of linear stochastic continuous-time dynamic systems represented by

$$\begin{aligned} &\mathbf{S}: \ dx = Axdt + Budt + \Gamma d\xi, \quad dz = Cxdt + d\eta \\ &\tilde{\mathbf{S}}: \ d\tilde{x} = \tilde{A}\tilde{x}dt + \tilde{B}\tilde{u}dt + \tilde{\Gamma}d\tilde{\xi}, \quad d\tilde{z} = \tilde{C}\tilde{x}dt + d\tilde{\eta} \end{aligned} \qquad (3)$$

where $x(t_0) = x_0$ and $\tilde{x}(t_0) = \tilde{x}_0$. The first equations in (3) are Ito stochastic differential equations describing the evolution of state vectors $x(t) \in R^n$ and $\tilde{x}(t) \in R^{\tilde{n}}$ of $\mathbf{S}$ and $\tilde{\mathbf{S}}$, respectively, driven by control inputs $u(t) \in R^m$ and $\tilde{u}(t) \in R^{\tilde{m}}$ (it is straightforward to connect model (2) with model (3)). Stochastic disturbances are modeled by Wiener processes $\xi(t) \in R^r$ and $\tilde{\xi}(t) \in R^r$ with incremental covariances $R_\xi dt$ and $R_{\tilde{\xi}} dt$, respectively. The second equations are the observation equations, where $\eta(t) \in R^q$ and $\tilde{\eta}(t) \in R^{\tilde{q}}$ are Wiener processes with incremental covariances $R_\eta dt$ and $R_{\tilde{\eta}} dt$, respectively. Vectors x_0 end $\tilde{x}_0$ are assumed to be Gaussian with means m_0 and $\tilde{m}_0$, and covariances R_0 and $\tilde{R}_0$, respectively. It is assumed that $\xi(t)$, $\eta(t)$ and x_0, as well as $\tilde{\xi}(t)$, $\tilde{\eta}(t)$ and $\tilde{x}_0$ are mutually independent. Matrices $A, B, \Gamma, C, \tilde{A}, \tilde{B}, \tilde{\Gamma}$ and $\tilde{C}$ are assumed to be constant. The basic assumption is that $n \le \tilde{n},\ p \le \tilde{p}$ and $q \le \tilde{q}$.

In general, for a stochastic process $\alpha(t)$ we shall denote the mean by $m_\alpha(t)$ and covariance by $R_\alpha(t_1, t_2)$. If $\alpha(t) = T\beta(t)$ $(\forall t \ge t_0)$, where $\alpha(t)$ and $\beta(t)$ are $n_\alpha -$ and $n_\beta -$ dimensional stochastic processes, respectively, and T a full rank matrix, we shall say that $\alpha(t)$ is a strong (strict) expansion of $\beta(t)$, i.e. $\alpha(t) = E_s[\beta(t); T]$, if $n_\alpha > n_\beta$, and that $\alpha(t)$ is a strong (strict) contraction of $\beta(t)$, i.e. $\alpha(t) = C_s[\beta(t); T]$, if $n_\alpha < n_\beta$. If, for the same processes, $m_\alpha(t) = Tm_\beta(t)$ and $R_\alpha(t_1, t_2) = TR_\beta(t_1, t_2)T^T$ $(\forall t, t_1, t_2 \ge t_0)$, we shall say that $\alpha(t)$ is a weak expansion of $\beta(t)$, i.e. $\alpha(t) = E_\omega[\beta(t); T]$ if $n_\alpha > n_\beta$, and a weak contraction, i.e. $\alpha(t) = C_\omega[\beta(t); T]$, if $n_\alpha < n_\beta$.

Definition 2.1 The system $\tilde{\mathrm{S}}$ includes the system S if there exist a quadruplet of full rank matrices $\{U_{n\times\tilde{n}}, V_{n\times\tilde{n}}, R_{\tilde{p}\times p}, S_{q\times\tilde{q}}\}$ satisfying $UV = I_n$, such that for any x_0 and $u(t)$ in S the conditions $\tilde{x}_0 = E_\omega[x_0; V]$ and $\tilde{u}(t) = E_s[u(t); R]$ imply $x(t) = C_\omega[\tilde{x}(t); U]$ and $z(t) = C_\omega[\tilde{z}(t); S]$ $(\forall t \ge t_0)$.

The expansion $\tilde{\mathrm{S}}$ contains all necessary information about S expressed in terms of second-order statistics, having in mind the Gauss-Markov properties of $x(t), \tilde{x}(t), z(t)$ and $\tilde{z}(t)$. Weak contractions/expansions are related to the states and outputs, and strong contractions/expansions to control inputs.

Restriction and aggregation represent two important special cases of inclusion.

Definition 2.2 The system S is restriction (type c, according to [11]) of the system $\tilde{\mathrm{S}}$ if there exist a full rank matrix $V_{\tilde{n}\times n}$ such that for any x_0 the conditions $\tilde{x}_0 = E_\omega[x_0; V]$ and $u(t) = C_s[\tilde{u}(t); Q]$ imply $\tilde{x} = E_\omega[x(t); V]$ and $\tilde{z}(t) = E_\omega[z(t); T]$ $(\forall t \ge t_0)$.

Theorem 2.1 The system S is restriction (type c) of $\tilde{\mathrm{S}}$ if there exist full rank matrices V, Q and T such that

$$\tilde{A}V = VA, \quad V\Gamma R_\xi \Gamma^T V^T = \tilde{\Gamma} R_{\tilde{\xi}} \tilde{\Gamma}^T, \ \tilde{B} = VBQ, \quad \tilde{C}V = TC, \quad TR_\eta T^T = R_{\tilde{\eta}}. \tag{4}$$

Definition 2. 3 The system S is an aggregation (type c) of $\tilde{\mathrm{S}}$ if there exist a triplet of full rank matrices (U, R, S) such that for any $\tilde{x}_0$ and $u(t)$ the conditions $x_0 = C_\omega[\tilde{x}_0; U]$ and $\tilde{u}(t) = E_s[u(t); R]$ imply $x(t) = C_\omega[\tilde{x}(t); U]$ and $z(t) = C_\omega[\tilde{z}(t); S]$ $(\forall t \ge t_0)$.

Theorem 2.2 The system S is an aggregation (type c) of $\tilde{\mathrm{S}}$ if there exist full rank matrices U, R and S such that

$$U\tilde{A} = AU, \quad \Gamma R_{\xi}\Gamma^{T} = U\tilde{\Gamma} R_{\tilde{\xi}}\tilde{\Gamma}^{T}U^{T},$$
$$U\tilde{B}R = B, \quad S\tilde{C} = CU, \quad SR_{\tilde{\eta}}S^{T} = R_{\eta} \tag{5}$$

3.2 INCLUSION OF ESTIMATORS

Consider time-invariant estimators $\mathbf{E}$ and $\tilde{\mathbf{E}}$ for $\mathbf{S}$ and $\tilde{\mathbf{S}}$, respectively,

$$\mathbf{E}: \quad d\omega = F\omega dt + Gudt + Ldz$$
$$\tilde{\mathbf{E}}: \quad d\tilde{\omega} = \tilde{F}\tilde{\omega}dt + \tilde{G}\tilde{u}dt + \tilde{L}d\tilde{z} \tag{6}$$

where $\omega(t) \in R^{s}$ and $\tilde{\omega}(t) \in R^{\tilde{s}}$ are the estimator outputs satisfying $s \le \tilde{s}$. State models for $\mathbf{S^e} = (\mathbf{S},\mathbf{E})$ and $\tilde{\mathbf{S}}^{\mathbf{e}} = (\tilde{\mathbf{S}},\tilde{\mathbf{E}})$ are, respectively,

$$\mathbf{S^e}: \quad dX = A^{e}Xdt + B^{e}udt + \Gamma^{e}d\Theta$$
$$\tilde{\mathbf{S}}^{\mathbf{e}}: \quad d\tilde{X} = \tilde{A}^{e}\tilde{X}dt + \tilde{B}^{e}\tilde{u}dt + \tilde{\Gamma}^{e}d\tilde{\Theta} \tag{7}$$

where $X = [x^{T}\omega^{T}]^{T}$, $\tilde{X} = [\tilde{x}^{T}\tilde{\omega}^{T}]^{T}$, $\Theta = [\xi^{T}\eta^{T}]^{T}$,

$$\tilde{\Theta} = \left[\tilde{\xi}^{T}\tilde{\eta}^{T}\right]^{T}, \; A^{e} = \begin{bmatrix} A & 0 \\ LC & F \end{bmatrix}; \; B^{e} = \begin{bmatrix} B \\ G \end{bmatrix}; \; \Gamma_{e} = \begin{bmatrix} \Gamma & 0 \\ 0 & L \end{bmatrix};$$

matrices $\tilde{A}^{e}, \tilde{B}^{e}$ and $\tilde{\Gamma}^{e}$ are defined analogous. It will be assumed that $X_0 = X(t_0)$ and $\tilde{X}_0 = \ddot{X}(t_0)$ are Gaussian with means M_0 and $\tilde{M}_0$ and covariances R_0^{X} and $R_0^{\tilde{X}}$, respectively.

Definition 2.4 The pair $(\tilde{\mathbf{S}},\tilde{\mathbf{E}})$ includes the par $(\mathbf{S},\mathbf{E})$ if $\tilde{\mathbf{S}}$ includes $\mathbf{S}$, and there exists a pair of full rank matrices $(D_{s\times\tilde{s}}, E_{\tilde{s}\times s})$ satisfying $DE = I_s$ such that for given X_0 and $u(t)$ the conditions $\tilde{X}_0 = E_{\omega}[X_0;V^{*}]$ and $\tilde{u}(t) = E_{s}[u(t);R]$ imply $X(t) = C_{\omega}[\tilde{X}(t);U^{*}]$ $(\forall t \ge t_0)$, where $U^{*} = diag\{U, D\}$ and $V^{*} = diag\{V, E\}$.

Theorem 2.3 The pair $(\mathbf{S},\mathbf{E})$ is a restriction (type c) of the pair $(\tilde{\mathbf{S}},\tilde{\mathbf{E}})$ if the conditions of Theorem 2.1 are satisfied, together with $\tilde{F}E = EF$, where E is full rank matrix, and $\tilde{G} = EGQ, \quad \tilde{L}T = EL$.

Theorem 2.4 The pair $(\mathbf{S},\mathbf{E})$ is a aggregation (type c) of the pair $(\tilde{\mathbf{S}},\tilde{\mathbf{E}})$ if the conditions of Theorem 2.2 are satisfied, together with $D\tilde{F} = FD$. Where D is full rank matrix, and $D\tilde{G}R = G, \quad D\tilde{L} = LS$.

3.3 CONTRACTIBILITY OF DYNAMIC CONTROLLERS

Let $\mathbf{S}^{f} = (\mathbf{S},\mathbf{E},\mathbf{F})$ and $\tilde{\mathbf{S}}^{f} = (\tilde{\mathbf{S}},\tilde{\mathbf{E}},\tilde{\mathbf{F}})$ where $\mathbf{F}$ and $\tilde{\mathbf{F}}$ are feedback mappings added to the pairs $(\mathbf{S},\mathbf{E})$ and $(\tilde{\mathbf{S}},\tilde{\mathbf{E}})$

$$\mathbf{F}: \quad u = K\omega + v; \quad \tilde{\mathbf{F}}: \quad \tilde{u} = \tilde{K}\tilde{\omega} + \tilde{v} \tag{8}$$

where K and $\tilde{K}$ are constant matrices, and v and $\tilde{v}$ reference signals. Obviously, we have

$$\mathbf{S}^f: \quad dX = A^* X dt + B^* v dt + \Gamma^* d\Theta$$
$$\tilde{\mathbf{S}}^f: \quad d\tilde{X} = \tilde{A}^* \tilde{X} dt + \tilde{B}^* \tilde{v} dt + \tilde{\Gamma}^* d\tilde{\Theta} \tag{9}$$

where

$$A^* = \begin{bmatrix} A & BK \\ LC & F + GK \end{bmatrix}, \quad B^* = \begin{bmatrix} B \\ G \end{bmatrix}, \quad \Gamma^* = \begin{bmatrix} \Gamma & 0 \\ 0 & L \end{bmatrix};$$

matrices $\tilde{A}^*, \tilde{B}^*$ and $\tilde{\Gamma}^*$ are defined analogously.

Definition 2.5 We say that the dynamic controller $(\tilde{\mathbf{E}}, \tilde{\mathbf{F}})$ for $\tilde{\mathbf{S}}$ is contractible to the dynamic controller $(\mathbf{E}, \mathbf{F})$ for $\mathbf{S}$ if $\tilde{\mathbf{S}}^f$ includes $\mathbf{S}^f$ in the sense of Definition 2.1.

Theorem 2.5 The controller $(\tilde{\mathbf{E}}, \tilde{\mathbf{F}})$ is contractible to the controller $(\mathbf{E}, \mathbf{F})$ when $(\mathbf{S}, \mathbf{E})$ is restriction (type c) of $(\tilde{\mathbf{S}}, \tilde{\mathbf{E}})$ and the condition $K = Q\tilde{K}E$ is satisfied.

Theorem 2.6 The controller $(\tilde{\mathbf{E}}, \tilde{\mathbf{F}})$ is contractible to the controller $(\mathbf{E}, \mathbf{F})$ when $(\mathbf{S}, \mathbf{E})$ is an aggregation (type c) of $(\tilde{\mathbf{S}}, \tilde{\mathbf{E}})$ and the condition $\tilde{K} = RKD$ is satisfied.

The above results show that K can be obtained for any given $\tilde{K}$ in the case of restriction, while L can be obtained from any given $\tilde{L}$ in the case of aggregation. When $F = A - LC$, $G = B, D = U$ and $E = V$ the corresponding explicit contraction mappings are $L = U\tilde{L}T$ and $K = Q\tilde{K}V$ [15].

3.4 INCLUSION OF PERFORMANCE INDICES

Consider the following pair of steady state performance indices for $\mathbf{S}$ and $\tilde{\mathbf{S}}$, respectively,

$$J(u) = \lim_{T \to \infty} \frac{1}{T} E\{\int_0^T (x^T W_x x + u^T W_u u) dt\}; \quad \tilde{J}(\tilde{u}) = \lim_{T \to \infty} \frac{1}{T} E\{\int_0^T (\tilde{x}^T \tilde{W}_x \tilde{x} + \tilde{u}^T \tilde{W}_u \tilde{u}) dt\} \tag{10}$$

where the matrices $W_x, W_u, \tilde{W}_x$ and $\tilde{W}_u$ are symmetric and positive semidefinite.

Definition 2.6 The pair $(\tilde{\mathbf{S}}, \tilde{\boldsymbol{J}})$ includes pair $(\mathbf{S}, \boldsymbol{J})$ in sense of the optimal feedback control law if the controller $(\tilde{\mathbf{E}}^*, \tilde{\mathbf{F}}^*)$ minimizing $\tilde{J}$ includes the controller $(\mathbf{E}^*, \mathbf{F}^*)$ minimizing J and

$$J(\mathbf{E}^*, \mathbf{F}^*) = \tilde{J}(\tilde{\mathbf{E}}^*, \tilde{\mathbf{F}}^*). \tag{11}$$

Theorem 2.7 If $\mathbf{S}$ is restriction (type c) of $\tilde{\mathbf{S}}$, then the pair $(\tilde{\mathbf{S}}, \tilde{\boldsymbol{J}})$ includes the pair $(\mathbf{S}, \boldsymbol{J})$ in the sense of the optimal feedback control law if

$$V^T M_x V = 0, \quad W_u^{-1} = Q\tilde{W}_u^{-1} Q^T, \tag{12}$$

where M_x is obtained from $\tilde{W} = U^T W_x U + M_x$. If $\mathbf{S}$ is an aggregation (type c) of $\tilde{\mathbf{S}}$, the pair $(\tilde{\mathbf{S}}, \tilde{\boldsymbol{J}})$ includes the pair $(\mathbf{S}, \boldsymbol{J})$ if

$$\tilde{W}_x = U^T W_x U; \quad W_u^{-1} = Q\tilde{W}_u^{-1} Q^T. \tag{13}$$

It follows that, if $\mathbf{S}$ is a restriction of $\tilde{\mathbf{S}}$ the optimal feedback gain matrix is contractible to the original space by $K = Q\tilde{K}V$.

3.5 OVERLAPPING DECENTRALIZED CONTROL

The essence of the application of the above exposed inclusion principle to the decentralized control design of systems with the overlapping structure $\mathbf{S}$, lies in the application of such an expansion which results into $\tilde{\mathbf{S}}$ in which subsystems of $\mathbf{S}$ appear as disjoint, e.g. [17, 11, 12, 14, 15]. For example, if $\mathbf{S}$ is defined by (3), where $A=[A_{ij}]$, $B=[B_{ij}]$, $C=[C_{ij}]$ $\Gamma=diag\{\Gamma_1,\Gamma_2,\Gamma_3\}$, $R_\xi=diag\{R_{\xi,1},R_{\xi,2},R_{\xi,3}\}$ and $R_\eta=diag\{R_{\eta,1},R_{\eta,2},R_{\eta,3}\}$, $(i,j=1,2,3)$, then we can consider, under certain conditions concerning submatrices $A_{13},A_{31},B_{13},B_{31},C_{13}$ and C_{31} in A,B and C, that it is composed of two overlapping subsystems $\tilde{\mathbf{S}}_1$ and $\tilde{\mathbf{S}}_2$ defined by system matrices $\tilde{A}^1=[A_{ij}]$, $\tilde{B}^1=[B_{ij}]$, $\tilde{C}^1=[C_{ij}]$, $\tilde{\Gamma}^1=diag\{\Gamma_1,\Gamma_2\}$, $\tilde{R}_\xi^1=diag\{R_{\xi,1}R_{\xi,2}\}$, $\tilde{R}_\eta^1=diag\{R_{\eta,1},R_{\eta,2}\}$, $(i,j=1,2)$ and $\tilde{A}_2=[A_{jk}]$, $\tilde{B}^2=[B_{jk}]$, $\tilde{C}^2=[C_{jk}]$, $\tilde{\Gamma}^2=diag\{\Gamma_2,\Gamma_3\}$, $\tilde{R}_\xi^2=diag\{R_{\xi,2},R_{\xi,3}\}$, $\tilde{R}_\eta^2=diag\{R_{\eta,2},R_{\eta,3}\}$, $(j,k=2,3)$, respectively. After performing an appropriate expansion and extracting the corresponding subsystems from $\mathbf{S}$, we shall look for decentralized dynamic controllers $(\tilde{\mathbf{E}}_1,\tilde{\mathbf{F}}_1)$ and $(\tilde{\mathbf{E}}_2,\tilde{\mathbf{F}}_2)$, characterized, in the case of local LQG optimal control, by the gain pairs $(\tilde{L}_1,\tilde{K}_1)$ and $(\tilde{L}_2,\tilde{K}_2)$, which, after being contracted back to the original space, result into a suboptimal controller $(\mathbf{E},\mathbf{F})$ for $\mathbf{S}$.

In the above context the main point is to find such pairs of matrices $(U,V),(Q,R)$ and $(S.T)$ which enable an expansion with satisfactory decoupling effects, as well as a direct contraction to the original space. We shall consider different restriction and aggregation relations between $\mathbf{S}$ and $\tilde{\mathbf{S}}$ obtained by using these matrices in two characteristic forms, e.g

$$V_1=\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix},\quad U_1=\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & \beta I & (1-\beta)I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \tag{14}$$

and $V_2=U_1^T$; $U_2=V_1^T$, where β is a scalar satisfying $0<\beta<1$; matrices R and T are analogous to V, while Q and S are analogous to U. Starting from matrices $A,B,C,\Gamma,R_\xi,R_\eta$, matrices $\tilde{A},\tilde{B},\tilde{C},\tilde{\Gamma},R_{\tilde{\xi}},R_{\tilde{\eta}}$ can be obtained by choosing e.g. matrix M_A in $\tilde{A}=VAU+M_A$, matrix M_B in $\tilde{B}=VBQ+M_B$, matrix M_C in $\tilde{C}=TCU+M_C$, etc. For example, conditions for both restriction and aggregation are satisfied for the following matrix $\tilde{A}$, obtained by using (U_1,V_1):

$$\tilde{A}=\begin{bmatrix} A_{11} & \beta A_{12} & (1-\beta)A_{12} & A_{13} \\ A_{21} & A_{22} & 0 & A_{23} \\ A_{21} & 0 & A_{22} & A_{23} \\ A_{31} & \beta A_{32} & (1-\beta)A_{32} & A_{33} \end{bmatrix}.$$

After expansion, the dynamic controller for the resulting $\tilde{\mathbf{S}}$ is designed by optimizing in the LQG sense separately $\tilde{\mathbf{S}}_1$ and $\tilde{\mathbf{S}}_2$, obtained by cutting $\tilde{A}$ adequately (as well as the remaining matrices in $\tilde{\mathbf{S}}$). The resulting estimator and feedback gain matrices $\tilde{L}_1,\tilde{L}_2,\tilde{K}_1$ and $\tilde{K}_2$ give $\tilde{L}_D=diag\{\tilde{L}_1,\tilde{L}_2\}$ and $\tilde{K}_D=diag\{\tilde{K}_1,\tilde{K}_2\}$, defining the overall controller $(\tilde{\mathbf{E}},\tilde{\mathbf{F}})$ for $\tilde{\mathbf{S}}$. The global performance index $\tilde{J}$ for $\tilde{\mathbf{S}}$ is constructed by using weighting matrices $\tilde{W}_x = diag\{\tilde{W}_x^1, \tilde{W}_x^2\}$ and $\tilde{W}_u = diag\{\tilde{W}_u^1, \tilde{W}_u^2\}$, where the local weighting matrices $\tilde{W}_x^1, \tilde{W}_x^2, \tilde{W}_u^1, \tilde{W}_u^2$ are chosen in accordance with (10), in order to satisfy inclusion of the performance indices $\tilde{J}$ and J. Contraction to the original space is done by $L = U\tilde{L}T$ and $K = Q\tilde{K}V$ after an eventual modification of either $\tilde{L}_D$ or $\tilde{K}_D$, aimed at satisfying contractibility conditions $(U\tilde{L} = U\tilde{L}TS$ or $Q\tilde{K} = Q\tilde{K}VU)$, having in mind that in the case of restriction we can never have a block-diagonal $\tilde{L}$, and case of aggregation a block-diagonal $\tilde{K}$. The resulting controller $(\mathbf{E}, \mathbf{F})$ is suboptimal with the suboptimality degree μ, i.e. $\mu^{-1} J^* = J$, where J^* is the minimal value of J corresponding to the globally optimal controller in the original space.

4. DECENTRALIZED LQG SUBOPTIMAL PLATOON CONTROL

Following the above exposed line of thought, a decentralized LQG suboptimal control strategy will be developed by considering a platoon of vehicles as a concatenation of overlapping “subsystems”. The i-th subsystems is defined by the following state model (see [13, 14] for the deterministic case)

$$\dot{\xi}_i = \begin{bmatrix} A_L & 0 \\ \overline{A}_d & A_v \end{bmatrix} \xi_i + \begin{bmatrix} B_L & 0 \\ 0 & B_v \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \end{bmatrix} + \begin{bmatrix} G_L & 0 \\ 0 & G_e \end{bmatrix} \begin{bmatrix} e_{i-1} \\ e_i \end{bmatrix} \tag{15}$$

where

$$A_L = \begin{bmatrix} 0 & -1 \\ 0 & -\alpha \end{bmatrix}, \quad \overline{A}_d^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad B_L^T = \begin{bmatrix} 0 & \alpha \end{bmatrix}, \quad G_L^T = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

and $\xi_i^T = [v_{i-1}\, a_{i-1}\, d_i\, v_i\, a_i]$. According to (2), the overlapping part in the state matrix is, obviously, A_L with both the preceding and the following subsystems. Having in mind the formalism of the Inclusion Principle, the above subsystems can be extracted from the basic model by expanding the state using a matrix V, which has, for two subsystems, the form (14), with appropriate dimensions (generalization to n vehicles is straightforward).

The above “subsystems” can hardly be given any precise physical interpretation; notice, however, that, formally, the noisy state vectors of the subsystems are supposed to be available in each vehicle [9]. The subsystems are not only state overlapping, but also input overlapping (they have one input in common), so that the input expansion is needed, as well; the corresponding transformation matrix R has form analogous to V. As u_i is essentially the physical control signal in the i-th vehicle, then u_{i-1} in the corresponding subsystem could be considered to represent, together with the corresponding part of the subsystem dynamics, the preceding part of the platoon, as viewed by i-th vehicle (for the second vehicle in the platoon this is exactly the leading vehicle dynamics). Therefore, u_i depends on the entire subsystem state, and u_{i-1} only on the part of the subsystem state vector overlapping with the preceding subsystem. After expansion, the subsystems in the platoon model appear as disjoint. Application of the LQG methodology based on the definition of local performance indices leads to local state feedback control (depending on the appropriate sets of measurements). Contraction to the original space provides a physically implementable control law.

As the leading vehicle dynamics represents formally a part of each subsystem, we shall describe the proposed control strategy consecutively, starting from the leading vehicle.

4.1 LEADING VEHICLE CONTROL

The leading vehicle is supplied with the reference command and uses its own state vector for control design. Formally, if the leading vehicle model is represented by

$$\dot{X}_L = A_L X_L + B_L u_1 + G_L e_1 \tag{16}$$

where $X_L^T = [v_1 \quad a_1]$, then the optimal feedback control law using noisy measurements $Y_L^T = \left[v_1 + n_1^v \quad a_1 + n_1^a\right]$ (where n_1^v and n_1^a are mutually independent white noise with variances r_1^v and r_1^a, respectively) should be found from the condition for the minimum of the performance index

$$J_L = E\left\{\int_{t_0}^{\infty}\left[(X_L - X_{1r})^T Q_L (X_L - X_{1r}) + R_L u_1^2\right]dt\right\} \tag{17}$$

where $X_{1r}^T = [v_r \quad a_r]$ is a time-varying reference supplied to the first vehicle, known entirely in advance, and $Q_L \geq 0$ end $R_L > 0$ are corresponding weights. This is, in fact, an LQG optimal tracking problem, which can be solved in the following way [1]:

$$\begin{aligned} &u_1 = -K_1 \hat{X}_L - M_1 X_{1r}, \quad K_1 = R_L^{-1} B_L^T P_L, \\ &M_1 = R_L^{-1} B_L^T (A_L - B_L K_1)^{-T} Q_L, \, P_L A_L + A_L^T P_L - P_L B_L R_L^{-1} B_L^T P_L + Q_L = 0 \end{aligned} \tag{18}$$

where $\hat{X}_L$ is obtained by the locally optimal Kalman filter obtained from (16). This control law is suboptimal, since the feedforward block is reduced to a constant matrix, this is, however, a very reasonable solution, having in mind characteristic forms of the reference command signals. A priori choice of the criterion weights can provide different tracking properties. Notice that the static steady state error reduces to zero, having in mind that A_L is singular [1].

4.2 GENERAL SUBSYSTEM CONTROL

Control of the second vehicle assumes that the leading vehicle control is appropriately designed. Consequently, control design for the general subsystem model (15) can be decomposed into two parts: first, u_{i-1} is found and the corresponding regulator is implemented and, second, u_i is found for the resulting system by using the complete feedback starting from the noisy state measurements. According to (18), we have

$$u_{i-1} = -K_1 [\hat{v}_{i-1} \quad \hat{a}_{i-1}]^T - M_1 X_{1r}. \tag{19}$$

After implementing (19), one comes to the following subsystem model

$$\dot{\xi}_i = \begin{bmatrix} A_L - B_L K_1 & 0 \\ \overline{A}_d & A_v \end{bmatrix} \xi_i + \begin{bmatrix} 0 \\ B_v \end{bmatrix} u_i + \begin{bmatrix} G_L & 0 \\ 0 & G_e \end{bmatrix} \begin{bmatrix} e_{i-1} \\ e_i \end{bmatrix} + \begin{bmatrix} K_1 \\ 0 \end{bmatrix} \varepsilon_{i-1} + \begin{bmatrix} -M_1 \\ 0 \end{bmatrix} X_{1r} \tag{20}$$

where ε_{i-1} is the estimation error for $[\hat{v}_{i-1} \quad \hat{a}_{i-1}]^T$ obtained by Kalman filter belonging to leading vehicle control law. Now, u_i is found from (20) by minimizing

$$J_i = E\left\{\int_{t_0}^{\infty}\left[(\xi_i - X_{2r})^T Q_i(\xi_i - X_{2r}) + R_i u_i^2\right]dt\right\}. \tag{21}$$

where $Q_i \geq 0$ and $R_i > 0$, while $X_{2r}^T = [d_r \ v_r \ a_r]$ is complete set of reference commands. The state weighting matrix is assumed to have the following specific form, coming out basically from the regulator structure adopted in [9]:

$$Q_i = \begin{bmatrix} p_1 & 0 & 0 & -p_1 & 0 \\ 0 & p_2 & 0 & 0 & -p_2 \\ 0 & 0 & q_{33} & 0 & 0 \\ -p_1 & 0 & 0 & q_{44}+p_1 & 0 \\ 0 & -p_2 & 0 & 0 & q_{55}+p_2 \end{bmatrix}. \tag{22}$$

In (22), q_{33} influences the spacing reference tracking, p_1 and p_2 influence tracking of the velocity and acceleration of the preceding vehicle, respectively, while q_{44} and q_{55} influence velocity and acceleration reference tracking. The problem posed belongs to the class of LQG optimal tracking problems with a priori known disturbances [1]. An approximately optimal solution, in the sense that all the gains are assumed to be constant, is given by

$$\begin{aligned} &u_i = -K_2\hat{X}_i - M_2 X_{2r} - M_3 X_{1r};\ K_2 = R_i^{-1}B_i^T P_2,\ M_2 = R_i^{-1}B_i^T(A_i - B_i K_2)^{-T} Q_i \\ &M_3 = R_i^{-1}B_i^T(A_i - B_i K_2)^{-T} P_2 B_M,\ P_2 A_i + A_i^T P_2 - P_2 B_i R_i^{-1} B_i^T P_2 + Q_i = 0 \end{aligned} \tag{23}$$

where

$$A_i = \begin{bmatrix} A_L - B_L K_1 & 0 \\ \overline{A}_d & A_v \end{bmatrix}, B_i^T = \begin{bmatrix} 0 & B_v \end{bmatrix}, B_M^T = \begin{bmatrix} -M_1 & 0 \end{bmatrix}$$

$\hat{X}_i$ represent the estimate of the subsystem state obtained by using the Kalman filter derived from (20), taking into account specific properties of the input disturbance. Notice that one disturbance term in (20) comes out from the first optimization step, i.e. from the optimal tracking problem solved by u_{i-1}. Consequently, M_2 represent the feedforward gain for the complete reference X_{2r}, while M_3 compensates the effects of the disturbance.

The state feedback gain $K_i^T = [K_1^T \quad K_2^T]$ has the LBT structure, in accordance with the information supposed to be locally available. The overall feedforward gain matrix M_i, which can be obtained simply from M_1, M_2 and M_3, multiplies essentially X_{2r}, since X_{1r} is subset of X_{2r}. It is important to notice that the steady-state error is again zero for constant references.

4.3 PLATOON CONTROL

Local regulators formulated for the subsystems are to be contracted to the original space before implementation. The state feedback gains are obtained according to paragraph 3.5 by using the transformation matrix Q analogous to U with $\beta = 0.5$, i.e. after contraction, one gets $K_M = Q\tilde{K}_i V$. The feed forward gains multiplying the reference signals are not contracted in accordance with the Inclusion Principle, since they are out of the feedback loop. The estimator gains are not contracted, as well, having in mind that all the local subsystem estimators remain uncontracted in the original system state space; formally, $D = E = I$ in terms of the inclusion of the estimators. The main additional requirement is here to keep the steady-state error at zero. It can be easily shown that the structure of M_2 and M_3 in (23) is such that the only nonzero elements are M_2^{51} and M_3^{51}; the only nonzero element in M_1 in (18) is M_1^{31}. It is possible to show that the required modification aimed at reducing the steady-state error to zero is to increment M_3^{51} in (23) by $\Delta M_3^{51} = -(A_K^{32} M_1^{31} + A_k^{35} M_3^{51}) / A_K^{35}$, where $A_K = (A_i - B_i K_M)^{-1}$ and $K_M^T = 0.5(K_2^T + [0 \quad 0 \quad 0 \quad K_1^T])$. The corresponding overall feedforward gain M_i (multiplying x_{2r}) contains only three nonzero elements: $M_i^{32} = M_1^{31}$, $M_i^{51} = M_2^{51}$ and $M_i^{52} = M_3^{51}$. The overall platoon tracks the command reference in a suboptimal way in the LQG sense, preserving the predefined information structure and ensuring the correct steady-state regime.

5. EXPERIMENTAL RESULTS

Numerous simulations have been undertaken; the platoon has been assumed to obey the nonlinear model (1) and control has been generated according to the described algorithm. Attention has been focused on the choice of the weights in (17) and (22) and noise influence. Figures 1 and 2 give time histories for a platoon of eight vehicles, containing velocities and inter–vehicle spacings; the first velocity and spacing plots correspond to a direct application of LQ feedback (not containing the estimators, [14]), while the second plots are obtained by using the whole proposed LQG suboptimal algorithm, including the local Kalman filters. The remaining design parameters have been $Q_L = diag\{200, 10\}$, $R_L = 10$, $p_1 = 100$, $p_2 = 50$, $q_{33} = 500$, $q_{44} = 300$ $q_{55} = 10$, $R_i = 10$, so that we obtained the following feedback and feed forward gains: $K_1 = [4.472 \quad 0.710]$, $K_2 = [-4.061 \; -1.258 \; -7.071 \; 6.728 \; 1.291]$, $M_1^{31} = 44.721$, $M_2^{51} = 26..672$, $M_3^{51} = -70.711$, (for $\alpha = 10$). Tracking capabilities and noise immunity of the proposed algorithm are obvious. Comparison with the results presented in [9] shows a substantial advantage of the proposed approach. It is to be noted that it is important to make decision about the relative importance of tracking the preceding vehicle velocity and the reference command, as well as about the weight of tracking the desired inter-vehicle spacing. The choice of the control weights influences the jerk level, which is important having especially in mind the introduced nonlinearities.

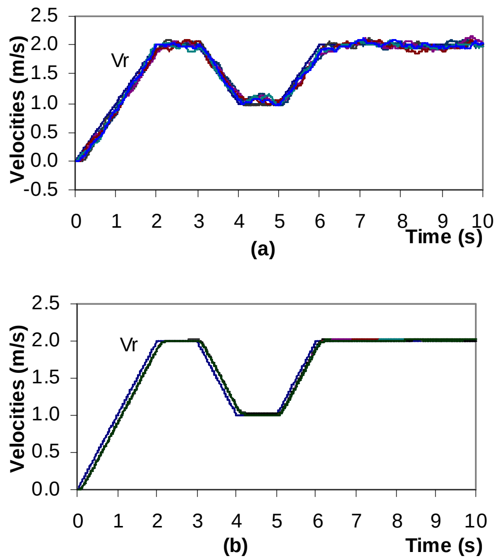

Figure 1 Velocities: (a) LQ and (b) LQG

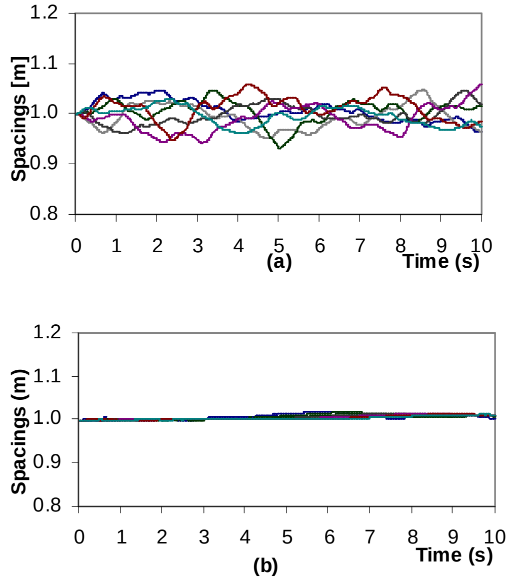

Figure 2 Spacing: (a) LQ and (b) LQG

6. CONCLUSION

In this paper the Stochastic Inclusion Principle has been applied to LQG suboptimal control of a platoon of automotive vehicles. Identification of input/state overlapping stochastic subsystems and their extraction by an appropriate expansion have lead to approximate LQG optimization, adapted to the LBT structure of the subsystem model. Simulation results show a high efficiency of the proposed algorithm, from the point of view of both tracking precision and noise immunity. One of the main problems for further investigations is the tracking precision in the case of long platoons.

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