Numerical Simulation of Non-Gaussian Random Fields

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Abstract

The non-Gaussian random fields are used to modelling some dynamic loads generated by wind turbulence, ocean waves, earthquake ground motion etc. These fields also represent the uncertain properties of different materials (reinforced concrete, composite, soils etc.).

This paper presents some methods and the corresponding algorithms to the numerical simulation of stationary non-Gaussian random fields characterized by power spectral density or equivalently autocorrelation function and by the marginal probability distributions. The considered methods include the generation of stationary Gaussian random fields based on the spectral representation theorem and their transformation of these fields in stationary non-Gaussian random fields.

Keywords: Stochastic processes, random fields, Gaussian and non-Gaussian random fields, translation processes and fields

1 Introduction and theoretical framework

The applications of random processes and fields theory in the area of engineering mechanics and structural engineering refer to random vibration problems concerning the random loads and are extended to stochastic structural and geotechnical systems considering the random material properties of different continuous media.

The important studies in the numerical simulation of random processes and fields were be done by Shinozuka and Deodatis [5], Yamazaki and Shinozuka [9], Grigoriu [2], Deodatis and Koutsourelakis [1], Xu and Graham-Brady [8] etc.

The theoretical framework [2, 3], of the next sections is presented in the following.

Let $X : I \times \Omega \to \mathbb{R}^d$ be a function of two arguments; $t \in I \subset \mathbb{R}$ (or $[0, \infty)$) and $\omega \in \Omega$, with $\{\Omega, F, P\}$ a probability space.

Definition 1. If $X(t) \in \mathbb{R}^d$ is a random vector on probability space $\{\Omega, F, P\}$, $(\forall)t \in I$, that is $X(t) \in F$, $(\forall)t \in I$, then X is said to be a ***d-dimensional real random process***, or ***vector stochastic process***. The function $X(., \omega)$ is called ***a sample path or a realization*** of X for a fixed ω. The function $X(t, .)$ is a random d-dimensional real vector for a fixed t.

Let $\{\Omega, F, P\}$ be a probability space and $X : D \times \Omega \to \mathbb{R}^d$, a function of two arguments, $\xi \in D \subset \mathbb{R}^q, q \geq 1$ is an integer and $\omega \in \Omega$.

Definition 2. If $X(\xi)$ is a random d-dimensional vector on the probability space $\{\Omega, F, P\}$, $(\forall)\xi \in D$, that is, $X(\xi) \in F, (\forall)\,\xi \in D$, then X is called ***random d-dimensional real field*** in D.

Definition 3. A stochastic process X is ***Gaussian*** if all its finite dimensional probability distributions are Gaussian, that is,

$$F_n(x^{(1)}, x^{(2)}, \ldots, x^{(n)}; t_1, t_2, ..., t_n) = P\left(\bigcap_{i=1}^{n}\left\{X(t_i) \in \prod_{k=1}^{d}(-\infty, x_k]\right\}\right),$$

where $x^{(i)} = (x_{i,1}, ..., x_{i,d}) \in \mathbb{R}^d,\ n \geq 1$, is Gaussian.

Definition 4. The process X is ***stationary in the strict sense***, or ***stationary***, if

$$(X(t_1), ..., X(t_n)) = (X(t_1 + \tau), ..., X(t_n + \tau)),$$

where $(\forall n \geq 1), t_i \neq t_j, i \neq j, i, j = \overline{1, n}, (\forall)\tau \in I$.

Remark 1. The marginal probability distributions of a random stationary process are invariant in time.

Definition 5. A process X is a ***translation process*** if its all coordinates are memoryless, measurable, nonlinear functions of a Gaussian d-dimensional process G, that is, $X_i(t) = g_i(G(t))$ and $g_i : \mathbb{R}^d \to \mathbb{R},\ i = \overline{1, d}$, are measurable functions, $(\forall)t \in I$.

Remark 2. The coordinates of X, $X_i(t) = g_i(G(t)), i = \overline{1, d},\ g_i : \mathbb{R} \to \mathbb{R}$ are, generally, real-valued functions. If the processes G_i are independent and $X_i(t) = g_i(G_i(t)),\ i = \overline{1, d}$, then the non-Gaussian processes X_i are independent of each other, otherwise the coordinates of X are dependent non-Gaussian processes.

Definition 6. ***The moments of order*** $q = \sum_{i=1}^{d} q_i$ of X are given by $\mu(q_1, ..., q_d) = E\left[\prod_{i=1}^{d} X_i^{q_i}\right]$, if the moments of order q_i of X_i exist and are finite.

Mean $\mu_i = E[X_i] = \mu(q_1, ..., q_d), q_i = 1, q_j = 0, i \neq j$.

Correlation of (X_i, X_j), $r_{i,j} = E[X_i X_j] = \mu(q_1, ..., q_d),\ q_i = q_j = 1,\ q_l = 0,\ l \neq i,\ l \neq j$.

Covariance of (X_i, X_j), $c_{i,j} = E[(X_i - \mu_i)(X_j - \mu_j)] = r_{i,j} - \mu_i \mu_j$.

Variance of X_i, $\sigma_i^2 = c_{i,i} = E\left[(X_i - \mu_i)^2\right] = r_{i,i} = \mu_i^2$.

Definition 7. The process X is defined as ***weakly stationary*** or ***stationary in the weak sense*** if:

1) The mean $\mu(t) = \mu = const$, is time invariant and

2) The correlation and the covariance functions $(r(t, t'), c(t, t'))$ depend only on the time lag $\tau = t - t'$, that is, $r(t, t') = r(\tau), c(t, t') = c(\tau)$.

Definition 8. Let $Y(t), t \in \mathbb{R}^{d'}$, be a $\mathbb{R}^d$- valued stationary Gaussian process $(d' = 1)$ or a homogeneous Gaussian field $(d' > 1)$. The argument of Y is time for stochastic processes or spatial coordinates for random fields. It is considered

that $Y = (Y_1, ..., Y_d), Y_i \longmapsto \mathcal{N}(0,1), i = \overline{1,d}$, $g : \mathbb{R}^d \to \mathbb{R}^d$ is a continuous function.

The function $X(t) = g(Y(t))$ is a non-Gaussian ***translation stochastic process*** or a ***translation random field***, $X_i(t) = g_i(Y_i(t)), i = \overline{1,d}$.

Examples. [2, 6, 7] **a) Gaussian fields.** Let $F_i, i = \overline{1,d}$ be some distribution functions, Φ denote the $\mathcal{N}(0,1)$ Gaussian distribution and $g_i = F_i^{-1} \circ \Phi$. Then $X_i \longmapsto F_i, i = \overline{1,d}$ and the correlation functions are

$$\rho_{i,j}(\tau) = E[X_i(t+\tau)X_j(t)] = \iint_{\mathbb{R}^2} g_i(u)g_j(v)\varphi(u,v,\rho_{i,j}(\tau))\,dudv,$$

where $\varphi(u,v,\rho_{i,j}(\tau))$ is density function of an $\mathbb{R}^2$-valued Gaussian variable with mean zero, variance one and correlation coefficient $\rho_{i,j}$.

b) Lognormal fields. If $Y(t)$ is a homogeneous, zero-mean, unit-variance Gaussian field, the random fields obtained by $X(t) = e^{\mu+\sigma Y(t)}$, are called *Lognormal fields* that are characterized by the one-dimensional marginal lognormal probability density function $f_X(x) = \dfrac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$.

c) Gamma fields. If Y_j is a collection of i.i.d. Gaussian fields with mean 0 and variance 1, then the random fields obtained via the transformation $X_m(t) = \frac{1}{2}\sum_{j=1}^{2m} Y_j^2(t)$ are called Gamma fields. That is because the corresponding one-dimensional marginal probability density function is a Gamma distribution with m degrees of freedom $f_{X_m}(t) = \frac{1}{\Gamma(m)} t^{m-1}e^{-t},\ t \geq 0$.

d) Beta fields. Given two independent Gamma fields, say X_m and X_n, characterized by same correlation coefficient, the random fields obtained via the transformation $X_{mn}(t) = \dfrac{X_m(t)}{X_m(t)+X_n(t)}$ are Beta fields, because their one-dimensional marginal probability density function is a $Beta(m,n)$ distribution: $f_{X_{mn}}(t) = \frac{1}{B(m,n)} t^{m-1}(1-t)^{n-1}, 0 \leq t \leq 1$.

Remark 3. a) The probability density function of X is

$$f_n(x_1, ..., x_n; t_1, ..., t_n) = \varphi(y_1, ..., y_n; \rho) \prod_{i=1}^{n} \frac{f(x_i)}{\varphi(y_i)},$$

where: $Y = (Y(t_1), ..., Y(t_n))$ is a Gaussian vector with mean $\mathbf{0}$ and covariance

$$\rho = \{\rho(t_i - t_j) = E[Y(t_i)Y(t_j)]\},\ x_i \in \mathbb{R},\ t_j \in \mathbb{R}^{d'},\ i = \overline{1,n},$$

$\varphi(y_1, ..., y_n)$ is the density probability function of Y, and $y_i = \Phi^{-1}(F(x_i))$.

b) The translation random processes and fields are stationary, can follow any marginal distribution and their correlation function is completely defined by marginal distribution F_i and the covariance functions $\rho_{i,j}$ of Y. Generally, the marginal distributions F_i and the correlation functions $\rho_{i,j}$ are available in applications, for example, they may be estimated from records of X.

Proposition 1. For $d = 1$, the translation process (field) has the marginal distribution function F.

$$P\left(X(t)\leq x\right)=P\left(\left(F^{-1}\circ\Phi\right)\left(Y(t)\right)\leq x\right)=P\left(F^{-1}\left(\Phi\left(Y(t)\right)\right)\leq x\right)=$$
$$P\left(Y(t)\leq\Phi^{-1}\left(F(x)\right)\right)=F(x).$$

From here follows Remark 3.b.

2 Numerical simulation of Gaussian random fields

2.1 Representation Theorems

Spectral representation theorem shows that each weakly stationary process can be approximate by a superposition of harmonics. These theorems can be considered to develop models for generating sample paths of stationary Gaussian processes, which consist of a superposition of harmonics with:

1) fixed frequencies and random amplitudes,

2) random frequencies and amplitudes.

Bochner's Theorem. [2] A continuous function $r:\mathbb{R}\to C$ is positive definite if and only if it has the representation $r(\tau)=\int e^{iv\tau}dS(v)$, where S is a real-valued, increasing and bounded function.

Remark 4. a) If X is a real-valued weakly stationary process, then its correlation function $r(\tau)=E\left[X(t+\tau)X(t)\right]$ is a positive definite function and thus there exists a real-valued, increasing and bounded function S, i. e. the equation in Bochner's theorem holds. The function S is called ***spectral distribution of*** X. If S is absolutely continuous, there exists a function $s(v)=\frac{dS(v)}{dv},v\in\mathbb{R}$, called ***spectral*** density or ***mean power spectral*** density of X.

b) Correlation function and spectral power density are Fourier pairs, that is, $r(\tau)=\int_{-\infty}^{\infty}e^{iv\tau}s(v)dv$ and $s(v)=\int_{-\infty}^{\infty}e^{-iv\tau}r(\tau)d\tau$.

Spectral representation theorem (complex case). If X is a complex-valued, weakly stationary and mean square continuous process $\left(\lim_{\tau\to0}E\left[\|X(t+\tau)-X(t)\|^2\right]=0\right)$ with spectral distribution S and spectral density s, then there is a complex-valued process with orthogonal increments Z such that

$$X(t)=\int_{-\infty}^{\infty}e^{ivt}dZ(v),\ (\forall)\,t\in C,Z(-\infty)=0,\ E\left[Z(t)\right]=0,$$
$$E\left[\left|Z(v)\right|^2\right]=S(v),\ E\left[\left|dZ(v)\right|^2\right]=dS(v)=s(v)dv$$

Spectral representation theorem (real case). If X is a real-valued, weakly stationary and mean square continuous process with spectral density s, then there are two real-valued processes with orthogonal increments U and V such that $X(t)=\int_0^{\infty}\left[\cos(vt)dU(v)+\sin(vt)dV(v)\right]$ and

$$E\left[U(v)\right]=E\left[V(v)\right]=0,\ E\left[dU(v)dV(v')\right]=0,$$
$$E\left[dU(v)^2\right]=E\left[dV(v)^2\right]=g(v)dv,\ (\forall)\,v,v'\geq0.$$

2.2 Simulation of Gaussian homogeneous random field using specified second order moments and fixed frequencies

Proposition 2. [2] If $\max_{1\leq p\leq n} v_p \to n \to \infty 0$, $\min_{1\leq k\leq d'} v_k^* \to \infty$, then

$$X^{(n)}(t) = \sum_{p=1}^{n} \left[A_p \cos(v_p t) + B_p \sin(v_p t)\right] \tag{1}$$

is a sample of X, where: $v_p \cdot t = \sum_{k=1}^{d'} v_{p,k} t_k$, $v_p = (v_{p,1}, ..., v_{p,d'})$, $t = (t_1, ..., t_{d'})$, $A_p, B_p \longmapsto \mathcal{N}(0,1)$ i.i.d., and X is a $\mathbb{R}^d$-valued random field, $X : D \times \Omega \to \mathbb{R}^d$, $D \subset \mathbb{R}^{d'}$, $d, d' \geq 1$, $D_1, ..., D_n$ is a partition of D, $v_p \in D_p$, interior point, generally D is a bounded rectangle centered at the origin of $\mathbb{R}^{d'}$, $D = \prod_{k=1}^{d'} [-v_k^*, v_k^*]$, $v_k^* > 0$.

Algorithm 1. ***The generation of a sample of random Gaussian homogeneous field*** X (Shinozuka method)[4, 5]

Step 1. Consider a cutoff frequency $v_k^* > 0, k = \overline{1, d'}$, $D = \prod_{k=1}^{d'} [-v_k^*, v_k^*]$, partition $D = \prod_{k=1}^{d'} [-v_k^*, v_k^*], v_k^* > 0$ in n rectangles and select frequencies v_p in theses rectangles, $p = \overline{1, n}$.

Step 2. Generate $A_p, B_p \longmapsto \mathcal{N}(0,1)$ i.i.d.

Step 3. Calculate $X^{(n)}(t)$ using (1), $t \in S \subset \mathbb{R}^{d'}$.

Particular case for $d' = 2$. We assume quadrant symmetry of spectral power density with respect to the origin. Let N_x and N_y be the number of points between 0 and v_1^*, v_2^*, $\Delta v_1 = \frac{v_1^*}{N_x}$, $\Delta v_2 = \frac{v_2^*}{N_y}$, $v_{1k} = k\Delta v_1$, $v_{2l} = l\Delta v_2$, $k = \overline{1, N_x}$, $l = \overline{1, N_y}$, M_x, M_y the number of points on the Ox, respectively Oy axes, $N_x < M_x, N_y < M_y$, $\Delta\xi_x = \frac{2\pi N_x}{v_1^* M_x}$, $\Delta\xi_y = \frac{2\pi N_y}{v_2^* M_y}$, $x_p = p\Delta\xi_x$, $p = \overline{0, M_x - 1}$, $y_q = q\Delta\xi_y$, $q = \overline{0, M_y - 1}$.

Thus, a sample of random Gaussian field is:

$$Y(x_p, y_q) = \sqrt{2} \sum_{k=1}^{N_x} \sum_{l=1}^{N_y} A_{kl} \left[\cos\left(v_{1k} x_p + v_{2l} y_q + \Phi_{kl}\right) + \cos\left(v_{1k} x_p - v_{2l} y_q + \Psi_{kl}\right)\right]$$

$$A_{kl} = \sqrt{2 s(v_{1k}, v_{2l}) \Delta v_1 \Delta v_2},\ k = \overline{1, N_x},\ l = \overline{1, N_y},$$

$$A_{kl} = 0 \text{ for } k = 0 \text{ or } l = 0,$$

Φ, Ψ i.i.d. uniform distributed on $[0, 2\pi]$.

The corresponding spectral density is

$$s(v_{1k}, v_{2l}) = \frac{1}{\Delta v_1 \Delta v_2} \left| \frac{1}{M_x M_y} \sum_{p=0}^{M_x - 1} \sum_{q=0}^{M_y - 1} Y(x_p, y_q) e^{-i(v_{1k} x_p + v_{2l} y_q)} \right|^2 .$$

This algorithm will be applied in the Section 4.

2.3 Simulation of Gaussian homogeneous random fields using random frequencies and amplitudes

Definition 8. [2] Let N be a Poisson process defined on $\mathbb{R}^{d'}$ with intensity $\lambda > 0$, T_k, $k = \overline{1,N}$, random points uniformly distributed on $\mathbb{R}^{d'}$, Y_k, $k = \overline{1,N}$, random variables i.i.d. with distribution function F, having zero mean and variance $\mu_2 = E\left[Y_k^2\right] < \infty$, $v = (v_1, ..., v_{d'}) \in D \subset \mathbb{R}^{d'}$, a bounded Borel subset of $\mathbb{R}^{d'}$, $D(T) = D \cap \left(\prod_{j=1}^{d'} [T_j, \infty)\right)$, $T = (T_1, ..., T_{d'})$, then $C(v) = \sum_{k=1}^{N(D)} Y_k \mathbf{1}_{D(T_k)}(v)$ is said a ***compound Poisson process***.

Proposition 3. [3] Let $C_p(v) = \sum_{k=1}^{N_p(D)} Y_{p,k} 1_{D(T_{p,k})}(v)$, $p = 1, 2$, be two compound Poisson processes, where N_p and $Y_{p,k}$ are independent copies of N and Y_k, respectively, and the random variable $T_{p,k}$ representing the random points in the definition of compound Poisson process C. If $\xi(v)^2 = \frac{s(v)}{\lambda \mu^2}$, $\lambda \to \infty$, and D approaches $\mathbb{R}^{d'}$, then

$$X^{(\lambda)}(t) = \sum_{k=1}^{N_1(D)} \xi(T_{1,k}) Y_{1,k} \cos(T_{1,k} t) - \sum_{k=1}^{N_2(D)} \xi(T_{2,k}) Y_{2,k} \sin(T_{2,k} t) \qquad (2)$$

becomes a sample of the homogeneous Gaussian field X.

Algorithm 2. ***The generation of a sample of random Gaussian homogeneous field*** X (Grigoriu method) [2]

Step 1. Select cutoff frequencies v_k^*, $k = \overline{1,d'}$, $D = \prod_{k=1}^{d'} [-v_k^*, v_k^*]$, $\lambda > 0$ the intensity of Poisson processes N_p, and a distribution function F of Y, such that $\xi(v)^2 = \frac{s(v)}{\lambda \mu^2}$.

Step 2. Generate samples of the Poisson points $\left(T_{p,1}, ..., T_{p,N_p(D)}\right)$, $p = 1, 2$, and the processes $\left(Y_{p,1}, ..., Y_{p,N_p(D)}\right)$, $Y_{p,k} \longmapsto F$ i.i.d.

Step 3. Calculate $X^{(\lambda)}(t)$ using (2), $t \in S \subset \mathbb{R}^{d'}$.

3 Algorithms for numerical simulation of translation non-Gaussian random fields

Yamazaki and Shinozuka [9]proposed an iterative method for generation a non-Gaussian stationary random field, having specified marginal probability distributions and spectral power density, using a Gaussian stationary random field. We consider a Gaussian field X, with zero mean and variance $\sigma_X^2 = \sigma^2$.

Algorithm 3. ***The generation of a sample of random non-Gaussian homogeneous field*** (Yamazaki-Shinozuka method)

Step 1. Input power spectral density s_X and marginal probability distribution F_X of non-Gaussian field X ; consider the Gaussian random field Y with mean zero, variance $\sigma_Y^2 = \sigma_X^2$ and spectral densitz $s_Y = s_X$. Generate the Gaussian random field $Y^{(1)}$; $X^{(1)} = g\left(Y^{(1)}\right)$, $\left(g = F_X^{-1} \circ \Phi\right)$; $k := 1$.

Step 2. Repeat

Calculate $s_Y^{(k)}$, $s_X^{(k)}$, $s_Y^{(k+1)} := \frac{s_X}{s_X^{(k)}} s_Y^{(k)}$

Generate $Y^{(k+1)}$ having the power spectral density $s_Y^{(k+1)}$;
Calculate $X^{(k+1)} := g\left(Y^{(k+1)}\right)$;
Until $s_X^{(k+1)} \approx s_X$.
Step 3. Return $X^{(k+1)}$ as a sample of X.

Deodatis and Koutsourelakis [1] modified this algorithm, considering an exponent α in the formula to correct the power spectral density, such that in step 2 of algorithm 3

$$s_Y^{(k+1)} := \left(\frac{s_X}{s_X^{(k)}}\right)^{\alpha} s_Y^{(k)} \tag{3}$$

They established by numerical experiments that $\alpha = 0.3$ gives a good convergence of algorithm.

Another modification in the algorithm 3 was introduced by Zerva [10] that calculates the spectral density function in $v_{1k} + \frac{\Delta v_1}{2}$, $v_{2l} + \frac{\Delta v_2}{2}$, $k = \overline{1, N_x}$, $l = \overline{1, N_y}$.

Xu and Graham-Brady [8] considered an algorithm similar to algorithm 3, but the equation to correct the spectral power density is

$$s_Y^{(k+1)} = s_X - s_X^{(k)} + s_Y^{(k)}, \tag{4}$$

for numerical simulation of random non-Gaussian processes.

Remark 5. In the ideal case when the power spectral density of non-Gaussian generated field is identical with the target, then power spectral density of Gaussian field don't modified for the next iterations and from this point of view the two equations for the correction of spectral density, (3) and (4), are equivalent.

In the algorithm proposed by Xu and Graham-Brady [8] the spectral densities $s_Y^{(k)}$, $s_X^{(k)}$ are not calculated for each sample of the processes Y, respectively X. The algorithm generates M samples of Y, $Y_1^{(k)}, ..., Y_M^{(k)}$ and consider $Y^{(k)} = \frac{1}{M}\sum_{j=1}^{M} Y_j^{(k)}$. The algorithm stops when $\left\| s_X - s_X^{(k)} \right\| < \epsilon$.

We apply this average to generate the non-Gaussian random fields in algorithm 4.

Algorithm 4. ***The generation of a sample of random non-Gaussian homogeneous field***

Step 1. Input the power spectral density s_X and the marginal distribution function F_X of the non-Gaussian field X; consider the Gaussian random field Y with mean zero, variance $\sigma_Y^2 = \sigma_X^2$ and spectral density $s_Y = s_X$; consider M the number of necessary samples to average. Generate Gaussian field $Y^{(1)}$; $X^{(1)} = g\left(Y^{(1)}\right)$, $\left(g = F_X^{-1} \circ \Phi\right)$; $k := 1$.
Step 2. Repeat
Calculate $s_Y^{(k)}$, $s_X^{(k)}$, $s_Y^{(k+1)}$ using (3) or (4);
For $j := 1 : M$ generate $Y_j^{(k+1)}$ having spectral density $s_Y^{(k+1)}$;
Calculate $Y^{(k+1)} = \frac{1}{M}\sum_{j=1}^{M} Y_j^{(k+1)}$; Calculate $X^{(k+1)} := g\left(Y^{(k+1)}\right)$;
$k := k + 1$;
Until $s_X^{(k+1)} \approx s_X$.
Step 3. Return $X^{(k+1)}$ as a sample of X.

4 Numerical application

We consider, as benchmark application a homogeneous two-dimensional non-Gaussian random field with Beta marginal probability density [9],

$$f(x) = \begin{cases} \dfrac{1}{B(q,p)} \dfrac{(x - x_{\min})^{q-1}(x_{\max} - x)^2}{(x_{\max} - x_{\min})^{q+p-1}}, & \text{if } x_{\min} \le x \le x_{\max} \\ 0, & \text{otherwise} \end{cases},$$

$F(x) = \frac{B_u(q,p)}{B(q,p)}$, $x_{\min} \le x \le x_{\max}$, and $B_u(q,p) = \int_0^u t^{q-1}(1-t)^{p-1}dt$, $u = \frac{x - x_{\min}}{x_{\max} - x_{\min}}$.

The mean and variance of the Beta distribution are $\mu = x_{\min} + \frac{q}{q+p}(x_{\max} - x_{\min})$,

$$\sigma^2 = \frac{qr\,(x_{\max} - x_{\min})^2}{(q+p)^2(q+p+1)}.$$

If the mean and the variance are known, it can obtain the range of variation of x, such that:

$$x_{\min} = \mu - \sigma\sqrt{\frac{q(q+p+1)}{p}},\ x_{\max} = \mu + \sigma\sqrt{\frac{p(q+p+1)}{q}}.$$

The spectral density $s(v_1, v_2) = \sigma^2 \frac{\delta^2}{4\pi} e^{-\frac{d^2(v_1^2 + v_2^2)}{4}}$ is quadrant symmetrical with respect to the origin, where: $\mu = 0$, $\sigma = 1$, $\delta = 1$, $q = 4$, $p = 2$, $\nu_1^* = \nu_2^* = 1.6\pi$, $N_x = N_y = 32$, $M_x = M_y = 64$.

Figures 4, 5, 6 and 7 present the results of simulation after five iterations (M5=), concerning the spectral density functions and samples of Gaussian and Beta fields obtained using the algorithm 4.

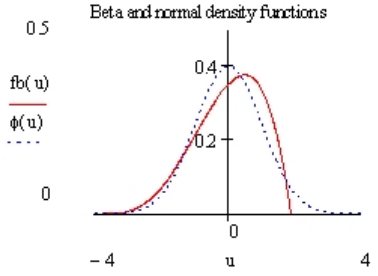

Figure 1

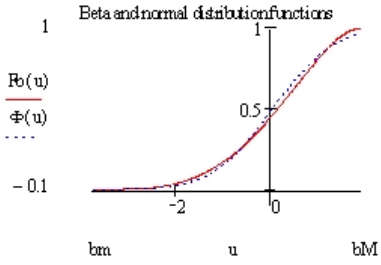

Figure 2

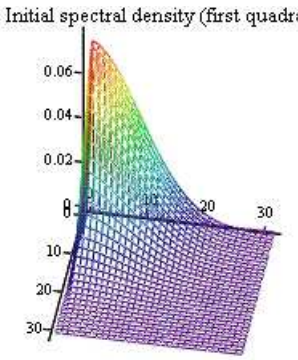

Figure 3

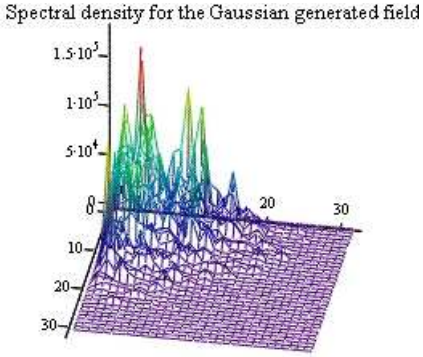

Figure 4

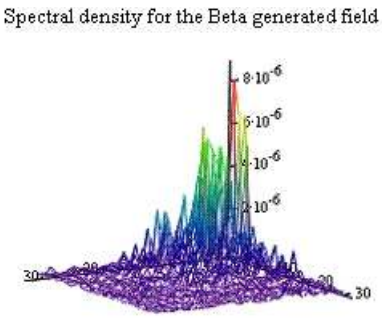

Figure 5

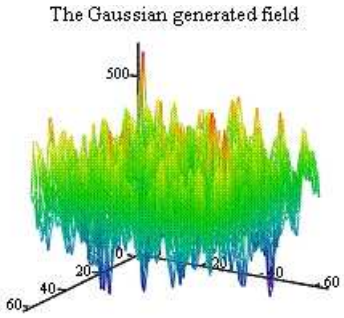

Figure 6

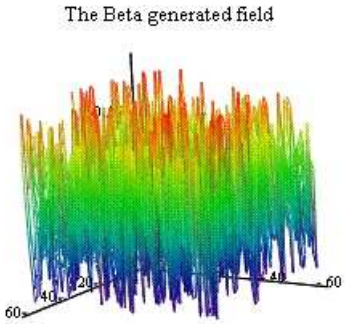

Figure 7

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