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ON MARITIME TRANSPORTATION PROBLEMS WITH INEXACT DATA

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Abstract *In this paper are presented the formulations of the fundamental maritime transportation problems and then is studied the case when restrictions are inclusions.*

In classical mathematical programming problems, in general, and in maritime transportation problems, in particular, we admit that we know exactly the coefficients which interfere in the problem. This assumption is not satisfied in most real problems because: the coefficients of the problem are submitted to measurement errors; the coefficients are not known exactly, but we known only that they belong to some convex sets.

Next, we will study the maritime transportation problem, in general, and then we will study the case when restrictions are inclusions.

In economical analysis of a voyage of a vessel Q, we have in view the following criterions: the reduction of voyage duration, the reduction of fuel consumption and avoidance exposing the vessel and equipments to excessive fraying or damages.

In maritime transportation problem there is the following axiom: “when the economical criterion is contradictory to vessel safety criterion, in choosing the route is priority the safety criterion”. This axiom is called the navigation principle.

Because the unit costs (per ton of transported merchandise) depends on: l - the type of merchandise; the distance and duration of voyage between the loading ports "i" and the unloading ports"j"; the type of the vessel"k", then they represent a quantity c_{lijk} characterized by 4 indices.

Therefore, the classical maritime transportation problem is the following:
“Let determine the merchandise quantities x_{lijk} (of type l) transported on the route $(i.j)$ with the vessel of type k such that the sum $\sum_{l=1}^{q}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{lijk}x_{lijk}$ to be minimal.”

The analytical form of this problem is:

$$\min \sum_{l=1}^{q}\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{p} c_{lijk} x_{lijk} \tag{1}$$

with conditions:

$$\begin{cases} \sum_{l=1}^{q}\sum_{k=1}^{p} x_{lijk} = a_{ij} \quad i=1,\dots,m,\ j=1,\dots,n\ , \\ \sum_{i=1}^{m}\sum_{k=1}^{p} x_{lijk} = b_{lj} \quad l=1,\dots,q,\ j=1,\dots,n\ , \\ \sum_{l=1}^{q}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{lijk} = q_k \quad k=1,\dots,q\ , \\ x_{lijk} \geq 0, \quad (\forall) l,i,j,k\ , \end{cases} \tag{2}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} = \sum_{l=1}^{q}\sum_{j=1}^{n} b_{jj} \tag{3}$$

where:

a_{ij} is the transported quantity of merchandise from port i to port j;

b_{lj} is the transported quantity of merchandise of type l in port j;

q_k is the total quantity of transported merchandise by the vessel of type k.

The condition (3) represents the equilibrium condition of the problem.

In this way, the classical maritime transportation problem is modeled by a transportation problem in equilibrium with minimum 4 indices. The number of indices may growth if is taken into account and others variables which can interfere into a concrete problem.

If one of four variables take a single value (i.e. unique loading port, unique unloading port, unique transported merchandise or unique vessel) then the transportation problem become a three-dimensional transportation problem studied by I.M.Stancu – Minasian [5].

There are more fundamentals types of maritime transportation problem. To give them the analytical form, we emphasize the next notations:

$i \in I$ - nominate the expedition port;

$j \in J$ - nominate the unloading port;

$k \in K$ - nominate the type of merchandise;

a_i - the entire quantity of merchandise loaded in port i;

c_k - the quantity of transported merchandises with the vessel of type k;

d_l - the quantities of transported merchandises of type l;

c_{ik} - the entire quantity of transported merchandise from port i with a vessel of type k;

c_{il} - the entire quantity of merchandise of type l loaded in port i;

d_{jk} - the entire quantity of merchandise unloaded in j by a vessel of type k;
e_{jl} - the entire quantity of merchandise of type l which is unloaded in port j;
q_{kl} - the entire quantity of merchandise of type l transported with a vessel of type k;
a_{ijk} - the entire quantity of transported merchandise from port i in port j with a vessel of type k;
b_{ikl} - the quantity of transported merchandise of type l from port i with a vessel of type k;
c_{ijl} - the quantity of transported merchandise of type l from port i to port j;
d_{jkl} - the quantity of merchandise of type l which arrived in port j, transported by the vessels of type k.

According to the manner of knowing some of the quantities presented, we can formulate the following fundamental maritime transportation problems:

If we know $c_{ijkl} \geq 0$; $a_{ijl} \geq 0$; $d_{jkl} > 0$ then the problem has the form:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} \sum_{l=1}^{q} c_{ijkl} x_{ijkl} \tag{4}$$

with restrictions:

$$\begin{cases} \sum_{l=1}^{q} x_{ijkl} = a_{ijk} \ , \ i \in I \ , \\ \sum_{j=1}^{n} x_{ijkl} = b_{ikl} \ , \ i \in I \ , \\ \sum_{k=1}^{p} x_{ijkl} = c_{ijl} \ , \ i \in I \ , \\ \sum_{i=1}^{m} x_{ijkl} = d_{jkl} \ , \\ x_{ijkl} \geq 0 \ , \end{cases} \tag{5}$$

and the equilibrium conditions:

$$\begin{cases} \sum_{k=1}^{p} a_{ijk} = \sum_{l=1}^{q} c_{ijl} = a_{ij} \ , \quad i \in I, \, j \in J \ , \\ \sum_{j=1}^{n} a_{ijk} = \sum_{l=1}^{q} b_{ikl} = b_{ik} \ , \quad i \in I, \, k \in K \ , \\ \sum_{k=1}^{p} b_{ikl} = \sum_{j=1}^{n} c_{ijl} = c_{il} \ , \quad i \in I, \, l \in L \ , \\ \sum_{i=1}^{m} a_{ijk} = \sum_{l=1}^{q} d_{jkl} = d_{jk} \ , \quad j \in J, \, k \in K \ , \\ \sum_{i=1}^{m} c_{ijl} = \sum_{j=1}^{n} d_{jkl} = e_{jl} \ , \quad k \in K, \, l \in L \ , \\ \sum_{i=1}^{m} b_{ikl} = \sum_{j=1}^{n} d_{jkl} = q_{kl} \ , \quad k \in K, \, l \in L \ , \\ \sum_{j=1}^{n} a_{ij} = \sum_{k=1}^{p} b_{ik} = \sum_{l=1}^{q} c_{il} = a_i \ , \quad i \in I \ . \end{cases} \tag{6}$$

The last relation from equilibrium conditions from (6) may be replaced by any of conditions:

$$\begin{cases} \sum_{i=1}^{m} a_{ij} = \sum_{k=1}^{p} d_{jk} = \sum_{l=1}^{q} c_{jl} = b_j \quad j \in J \ , \\ \sum_{i=1}^{m} b_{ik} = \sum_{j=1}^{n} d_{jk} = \sum_{l=1}^{q} q_{kl} = c_k \quad \mathrm{k} \in K \ , \end{cases} \tag{7}$$

$$\begin{cases} \sum_{i=1}^{m} c_{ij} = \sum_{j=1}^{n} e_{jl} = \sum_{k=1}^{p} q_{kl} = d_l \quad l \in L \ , \\ \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{p} c_k = \sum_{l=1}^{q} d_l = \mathrm{T} \ . \end{cases} \tag{7'}$$

This transportation problem is called the tetraxial model of the transportation problem.

If in the transportation problem is know the quantities $c_{ijkl} \geq 0$, $a_{ij} > 0$, $b_{jk} > 0$, $c_{il} > 0$, $d_{jk} > 0$, $e_{jl} > 0$, $q_{kl} > 0$, then the transportation problem has the form:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} \sum_{l=1}^{q} c_{ijkl} x_{ijkl} \tag{8}$$

with restrictions:

$$\begin{cases} \sum_{k=1}^{p}\sum_{l=1}^{q} x_{ijkl} = a_{ij} \ , \quad i \in I,\ j \in J \ , \\ \sum_{j=1}^{n}\sum_{l=1}^{q} x_{ijkl} = b_{ik} \ , \quad i \in I,\ k \in K \ , \\ \sum_{j=1}^{n}\sum_{k=1}^{p} x_{ijkl} = c_{il} \ , \quad i \in I,\ l \in L \ , \\ \sum_{i=1}^{m}\sum_{l=1}^{q} x_{ijkl} = d_{jk} \ , \quad j \in J,\ k \in K \ , \\ \sum_{i=1}^{m}\sum_{k=1}^{p} x_{ijkl} = e_{jl} \ , \quad j \in J,\ l \in L \ , \\ \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijkl} = q_{kl} \ , \quad k \in K,\ l \in L \ , \\ \qquad x_{ijkl} \ge 0 \ , \quad i \in I,\ j \in J,\ k \in K,\ l \in L \ , \end{cases} \tag{9}$$

and the equilibrium conditions:

$$\begin{cases} \sum_{j=1}^{n} a_{ij} = \sum_{k=1}^{p} b_{ik} = \sum_{l=1}^{q} c_{il} = a_i \ , \quad i \in I \ , \\ \sum_{i=1}^{m} a_{ij} = \sum_{k=1}^{p} d_{jk} = \sum_{l=1}^{q} e_{jl} = b_j \ , \quad j \in J \ , \\ \sum_{i=1}^{m} b_{ik} = \sum_{j=1}^{n} d_{jk} = \sum_{l=1}^{q} q_{kl} = c_k \ , \quad k \in K \ , \\ \sum_{i=1}^{m} c_{il} = \sum_{j=1}^{n} e_{jl} = \sum_{k=1}^{p} q_{kl} = d_l \ , \quad l \in L \ , \\ \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{p} c_k = \sum_{l=1}^{q} d_l = \mathrm{T} \ . \end{cases} \tag{10}$$

Knowing others quantities which interfere in the maritime transportation problem, we can formulate another 16 linear maritime transportation problem with four indices.

Next, we will consider a simplified model of the previous model, namely we will suppose that we have a single type of merchandise which is transported and a single type of vessel which make the transport.

Let be the next maritime transportation problem which is the classical transportation problem:

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,\ldots,n, \tag{11}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,\ldots,m,$$

$$x_{ij} \geq 0 \; ; \quad \sum_{j} b_j = \sum_{i} a_i \; ; \quad i = 1,\ldots,m, \; j = 1,\ldots,n \quad ,$$

where:

x_{ij} is the merchandise volume which can be transported from the contractor i to beneficiary j ;

c_{ij} - the transportation costs;

a_i - the volume which can be delivered by i ;

b_j - the volume which is requested by j .

Unlike this problem, the transportation problem with inclusions restrictions has the following form:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} \subseteq A_i, \; i = 1,\ldots,m, \tag{12}$$

$$\sum_{i=1}^{n} x_{ij} \subseteq B_j, \; j = 1,\ldots,n,$$

$$x_{ij} \geq 0, \quad i = 1,\ldots,m, \quad j = 1,\ldots,n \quad ,$$

where $A_i \; \left(i = 1,\ldots,m\right)$ and $B_j \; \left(j = 1,\ldots,n\right)$ are data sets.

Next, we will assume that

$$A_i = \left[\underline{a}_i, \overline{a}_i\right] \text{ and } B_j = \left[\underline{b}_j, \overline{b}_j\right] ,$$

where $\underline{a}_i = \inf A_i$, $\overline{a}_i = \sup A_i$ and $\underline{b}_j = \inf B_j$, $\overline{b}_j = \sup B_j$.

Hence, the transportation problem with inexact data (with inclusions restrictions) can be state in this way:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} \subseteq \left[\underline{a}_i, \overline{a}_i\right], \; i = 1,\ldots,m, \tag{13}$$

$$\sum_{i=1}^{m} x_{ij} \subseteq \left[\underline{b}_j, \overline{b}_j\right], \; j = 1,\ldots,n,$$

$x_{ij} \geq 0,\ i = 1,\ldots,m,\ j = 1,\ldots,n$.

Obviously, this problem can be transformed into an equivalent linear programming problem, but it is more conveniently to find an equivalent maritime transportation problem. This equivalent problem expressed in terms of contractors, beneficiaries, goods quantities (marked over the arcs from figure) and the transportation costs (written below of the arcs from figure) is represented in Figure 1.

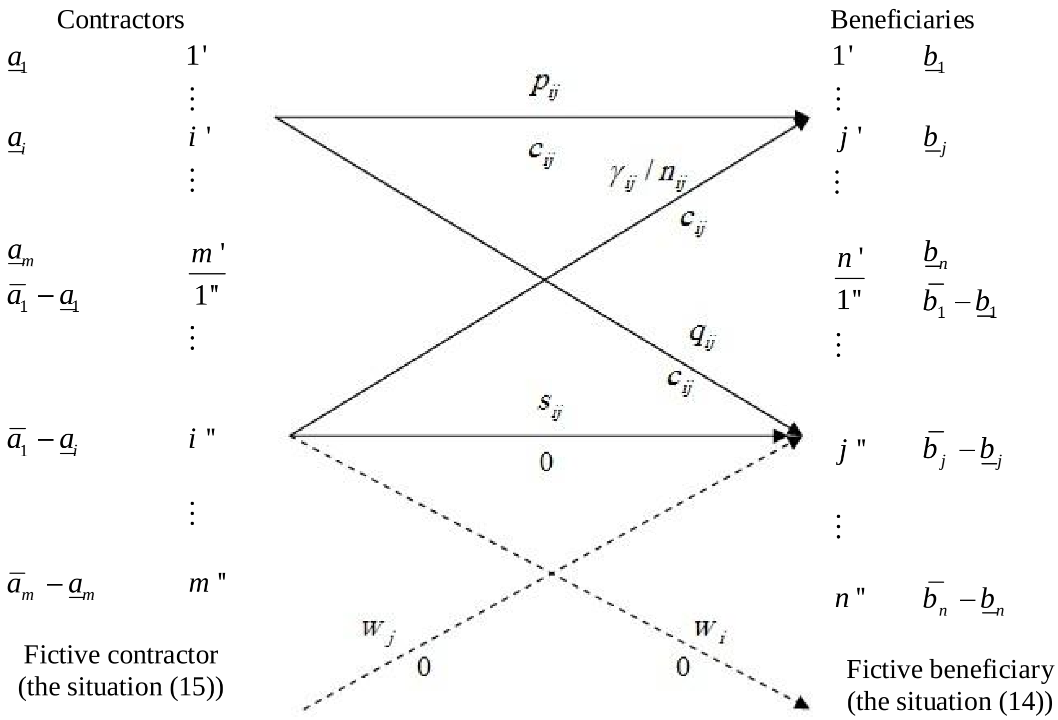

Figure 1

Lets remark that each stock building i can be divided into two auxiliary stock buildings i' and i'', which dispose of the quantities $\underline{a}_i$, and $\overline{a}_i - \underline{a}_i$ respectively. Analogue, each beneficiary j can be divided into j' and j'', which are needed the quantities $\underline{b}_j$, and $\overline{b}_j - \underline{b}_j$ respectively.

Now, we make the following notations:

$$\sum_{i=1}^{m} \underline{a}_i = \underline{A}, \quad \sum_{i=1}^{m} \overline{a}_i = \overline{A}, \quad \sum_{j=1}^{n} \underline{b}_j = \underline{B}, \quad \sum_{j=1}^{n} \overline{b}_j = \overline{B}.$$

There are possible two situations:

$$\underline{A} \leq \overline{B} \leq \overline{A} \quad , \tag{14}$$

$$\underline{B} \leq \overline{A} \leq \overline{B} \quad . \tag{15}$$

In situations (14), there is an overproduction $\bar{A}-\bar{B}$ which is orientated to a fictive beneficiary. In situation (15), there is unsatisfied request, which is covered by a fictive contractor. Obviously, the fictive contractor, respectively the fictive beneficiary does not exist concretely, the respective merchandise quantity remains, in fact, in stock buildings.

Let consider now the case of the overproduction (14). The dimension of the problem is now $2m \times (2n+1)$. The transported quantities and the transportation costs, written in round brackets, are given in table 1.

	$1 \; \dots \; n$	$n+1 \; \dots \; 2n$	$2n+1$	$\sum_{j=1}^{2n+1}$
$1 \; \vdots \; m$	$p_{ij}\,(c_{ij})$	$q_{ij}\,(c_{ij})$	$t_i = 0$ (∞)	$\underline{a}_i$
$m+1 \; \vdots \; 2m$	$\gamma_{ij}\,(c_{ij})$	$s_{ij}\,(0)$	$w_i\,(0)$	$\bar{a}_i - \underline{a}_i$
$\sum_i$	$\underline{b}_j$	$\bar{b}_j - \underline{b}_j$	$\bar{A}-\bar{B}$	$\bar{A}$

Table 1

The meaning of the measures s_{ij}, w_i and t_i are:

$\sum_{j=1}^{n} s_{ij}$ - the merchandise quantity delivered by the contractor i,

$\sum_{i=1}^{m} s_{ij}$ - the quantity which the beneficiary j does not receive it in relation with $\bar{b}_j$,

w_i - the overproduction of the contractor i,

$\sum_{i=1}^{m} s_{ij} = \bar{A}-\bar{B}$ and t_i are the artificially variables inserted for uniformity of the description.

In case (15) of over request, the problem dimension is $(2m+1)\times 2$. The transported quantities and the transportation costs are given in table 2.

	$1 \; \dots \; n$	$n+1 \; \dots \; 2n$	$\sum_{i=1}$
$1 \; \vdots \; m$	$p_{ij}\,(c_{ij})$	$q_{ij}\,(c_{ij})$	$\underline{a}_i$
$m+1 \; \vdots \; 2m$	$n_{ij}\,(c_{ij})$	$s_{ij}\,(0)$	$\bar{a}_i - \underline{a}_i$

$2m+1$	$t_i = 0$ (∞)	$w_i\,(0)$	$\bar{B} - \bar{A}$
$\sum_j$	$\underline{b}_j$	$\bar{b}_j - \underline{b}_j$	$\bar{B}$

Table 2

The meanings of s_{ij} and t_j are the same as in the previous case, while w_j is the uncovered request, i.e. the request satisfied by the fictive contractor, $\sum_{j=1}^{n} w_j = \bar{B} - \bar{A}$.

Thus, the transported merchandise quantities from contractor i to beneficiary j are the following:

a) in case (14): $x_{ij} = p_{ij} + q_{ij} + \gamma_{ij}$, (16)

b) in case (15): $x_{ij} = p_{ij} + q_{ij} + n_{ij}$. (17)

CONCLUSIONS

The state of the transportation problem with inexact data presented above has certainly advantages both theoretically and practically. This state is not too different of the classical transportation problem, which means in this case that can be used classical algorithms from transportation problem. More, the classical transportation problem is a particular case of the problem presented above, precisely when $\underline{a}_i = \bar{a}_i$ and $\underline{b}_j = \bar{b}_j$, for $\forall i, j$.

From practical point of view, this approach of the transportation problem can be preferred to the classical approach.

Also, the rise of the problem dimensions is not too important, because the problem can be solved with the same methods of the classical transportation problem.

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